MATHEMATICS

Chapter 9: Circles

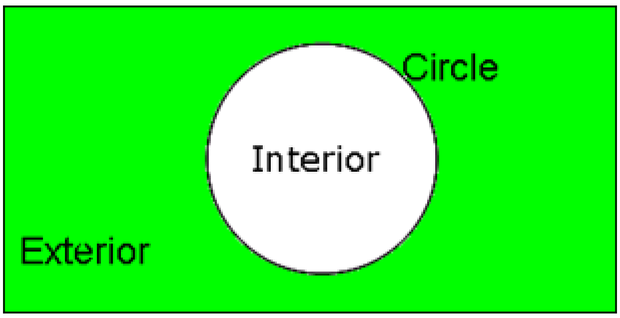
Circles

Circle

- The set of all the points in a plane that is at a fixed distance from a fixed point makes a circle.
- A Fixed point from which the set of points are at fixed distance is called the centre of the circle.
- A circle divides the plane into 3 parts: interior (inside the circle), the circle itself and exterior (outside the circle)

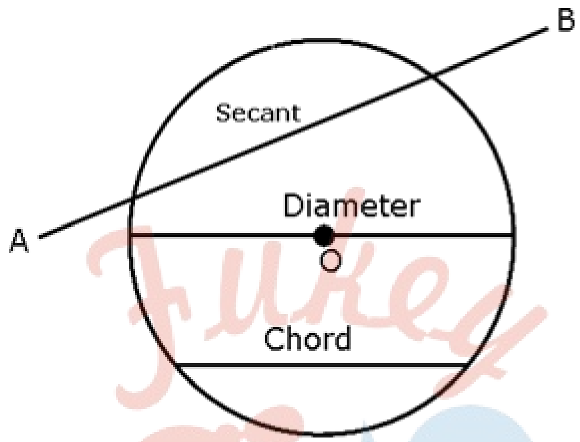
Division of a plane using circle

- A circle divides the plane on which it lies into three parts: inside the circle, the circle and outside the circle.
- All the points lying inside a circle are called its **interior points** and all those points which lie outside the circle are called its **exterior points**.
- The collection (set) of all interior points of a circle is called the **interior of the circle** while the collection (set) of all exterior points of a circle is called the **exterior of the circle**.



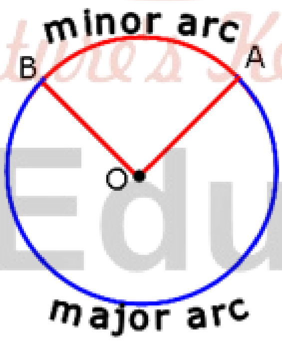
Chord, diameter and secant of a circle

- A line can meet a circle at the most in two points and the line segment joining two points on acircle is called a **chord** of the circle.
- A chord passing through the center of the circle is called a **diameter** of the circle. A diameter of the circle is its longest chord. It is equal to two times the radius.
- A line which meets a circle in two points is called a **secant** of the circle.



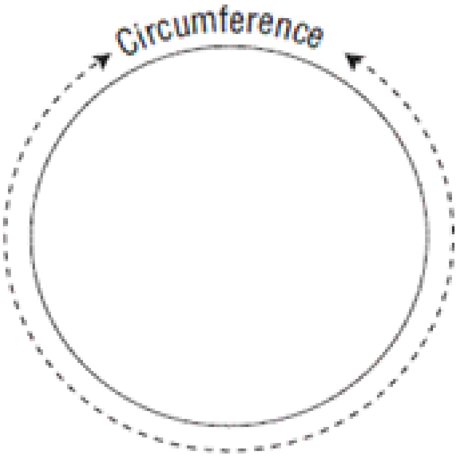
Arc of the circle

- A (continuous) part of a circle is called an **arc** of the circle. The arc of a circle is denoted by the symbol ' '.
- When an arc is formed, it divides the circle into two pieces (between the points A and B), the smaller one and the longer one. The smaller one is called the **minor arc** of the circle, and the greater one is called the **major arc** of the circle.

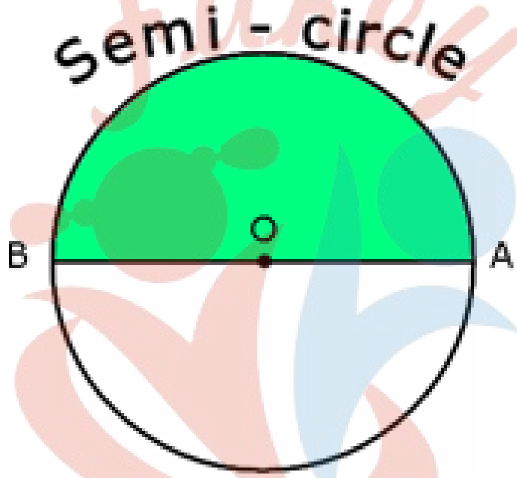


Circumference and Semi-circle

- The length of the complete circle is called the **circumference** of the circle.



- One-half of the whole arc (circumference) of a circle is called a **semi-circle**.



Central angle and Degree measure

- Any angle whose vertex is centre of the circle is called a **central angle**.
- The **degree measure of a minor arc** is the measure of the central angle subtended by an arc.
- The degree measure of a circle is 360°. The degree measure of a semi-circle is 180° (half of the circle).
- The degree measure of a major arc is (360° - θ°), where θ° is the degree measure of the corresponding minor arc.

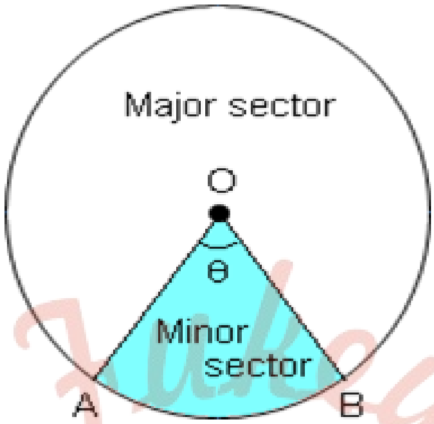
Congruent circles and arcs

- Two **circles** are said to be **congruent** if and only if either of them can be superposed on the other so as to cover it exactly.
- Accordingly, two **arcs** of a circle (or of congruent circles) are **congruent** if either of them can be superposed on the other so as to cover it exactly.

Sector of a circle

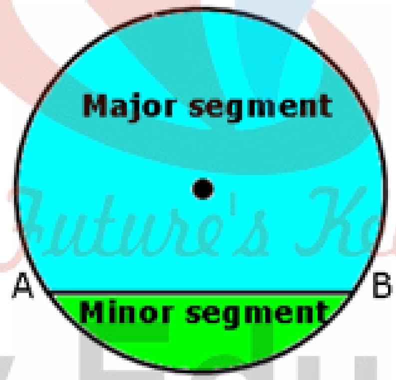
- The part of the plane region enclosed by an arc of a circle and its two bounding radii is called **sector** of a circle.

- If the central angle of a sector is more than180°, then the sector is called a **major sector** and if the central angle is less than 180°, then the sector is called a **minor sector**.



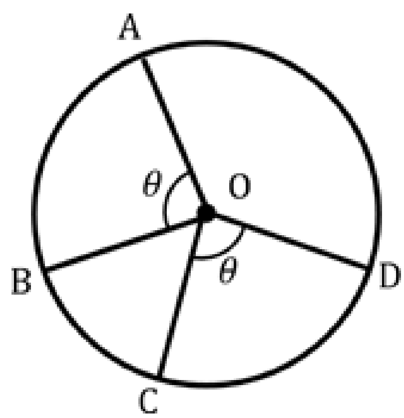
Segment of a circle

- A chord of a circle divides it into two parts. Each part is called a **segment** of the circle.
- The part containing the minor arc is called the **minor segment**, and the part containing the major arc is called the **major segment**.

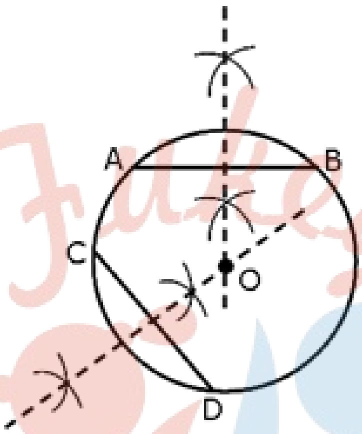


Angle subtended by a chord and perpendicular drawn to a chord

- Equal chords of a circle subtend equal angles at the centre.

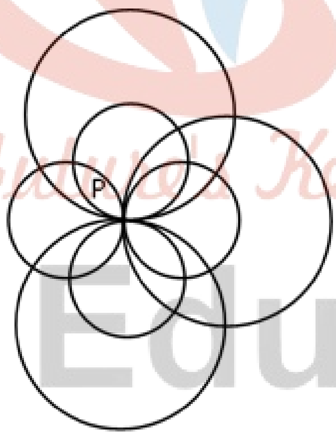


- If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.
- In a circle, perpendicular from the center to a chord bisects the chord.
- A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
- Perpendicular bisectors of two chords of a circle, intersect each other at the centre of the circle.

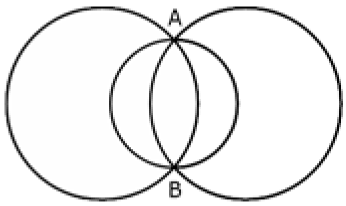


Number of circle through one or more point(s)

- An infinite number of circles can be drawn through a given point, say P.



- An infinite number of circles can be drawn through two given points, say A and B.



- One and only one circle can be drawn through three non-collinear points.

Distance of chord from the centre

- The length of the perpendicular from a point to a line is the distance of the line from the point.

- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).
- Chords equidistant from the centre of a circle are equal in length.

Angle subtended by an Arc of a circle

- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- If two chords of a circle are equal, then their corresponding arcs are congruent.
- Conversely, if two arcs are congruent, then their corresponding chords are equal.
- Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

Con-cyclic points

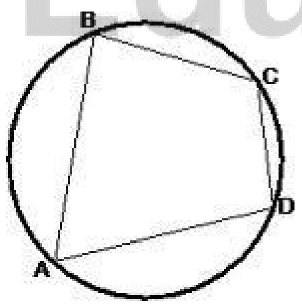
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points are con-cyclic, i.e., they lie on the same circle.
- Angles in the same segment of a circle are equal.

Angle in a semi-circle

- An angle in a semi-circle is a right angle.
- The arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

Cyclic quadrilaterals

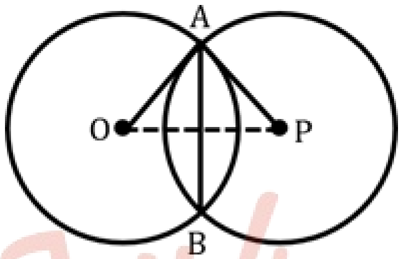
A quadrilateral, all the four vertices of which lie on a circle is called a **cyclic quadrilateral**. The four vertices A, B, C and D are said to be concyclic points.



Properties of cyclic quadrilateral

- The opposite angles of a cyclic quadrilateral are supplementary i.e. their sum is 180°.
- If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

- Any exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- The quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.
- The line of centres of two intersecting circles subtends equal angles at the two points of intersection.

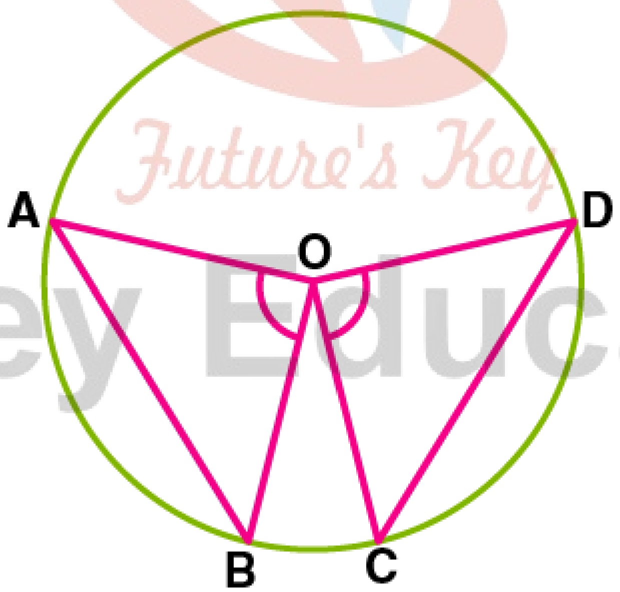


In the figure, angle OAM = angle PAM.

- If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, then it is a rectangle.
- If the non-parallel sides of a trapezium are equal, then it is cyclic.

Theorem of equal chords subtending angles at the centre.

Equal chords subtend equal angles at the centre.



Proof: AB and CD are the 2 equal chords.

In Δ AOB and Δ COD

OB = OC [Radii]

OA = OD [Radii]

AB = CD [Given]

ΔAOB ≅ ΔCOD (SSS rule)

Hence, ∠AOB = ∠COD [CPCT]

Theorem of equal angles subtended by different chords.

If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Proof: In ΔAOB and ΔCOD

OB = OC [Radii] ∠AOB=∠COD [Given]

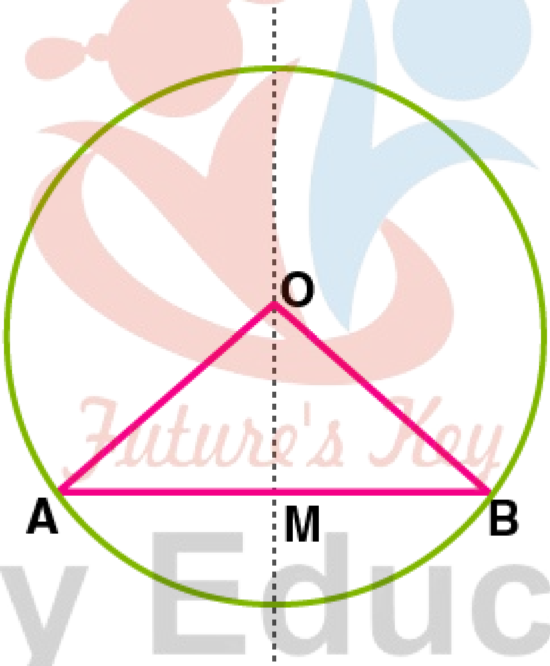
OA = OD [Radii]

ΔAOB ≅ ΔCOD (SAS rule)

Hence, AB=CD [CPCT]

Perpendicular from the centre to a chord bisects the chord.

Perpendicular from the centre of a circle to a chord bisects the chord.



Proof: AB is a chord and OM is the perpendicular drawn from the centre.

From ΔOMB and ΔOMA,

∠OMA = ∠OMB = 90^0 OA = OB (radii)

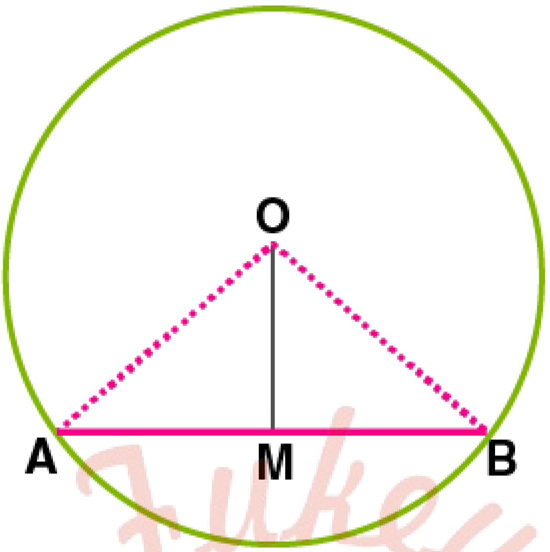
OM = OM (common)

Hence, ΔOMB ≅ ΔOMA (RHS rule)

Therefore AM = MB [CPCT]

A Line through the centre that bisects the chord is perpendicular to the chord.

A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



Proof: OM drawn from the center to bisect chord AB.

From ΔOMA and ΔOMB,

OA = OB (Radii)

OM = OM (common)

AM = BM (Given)

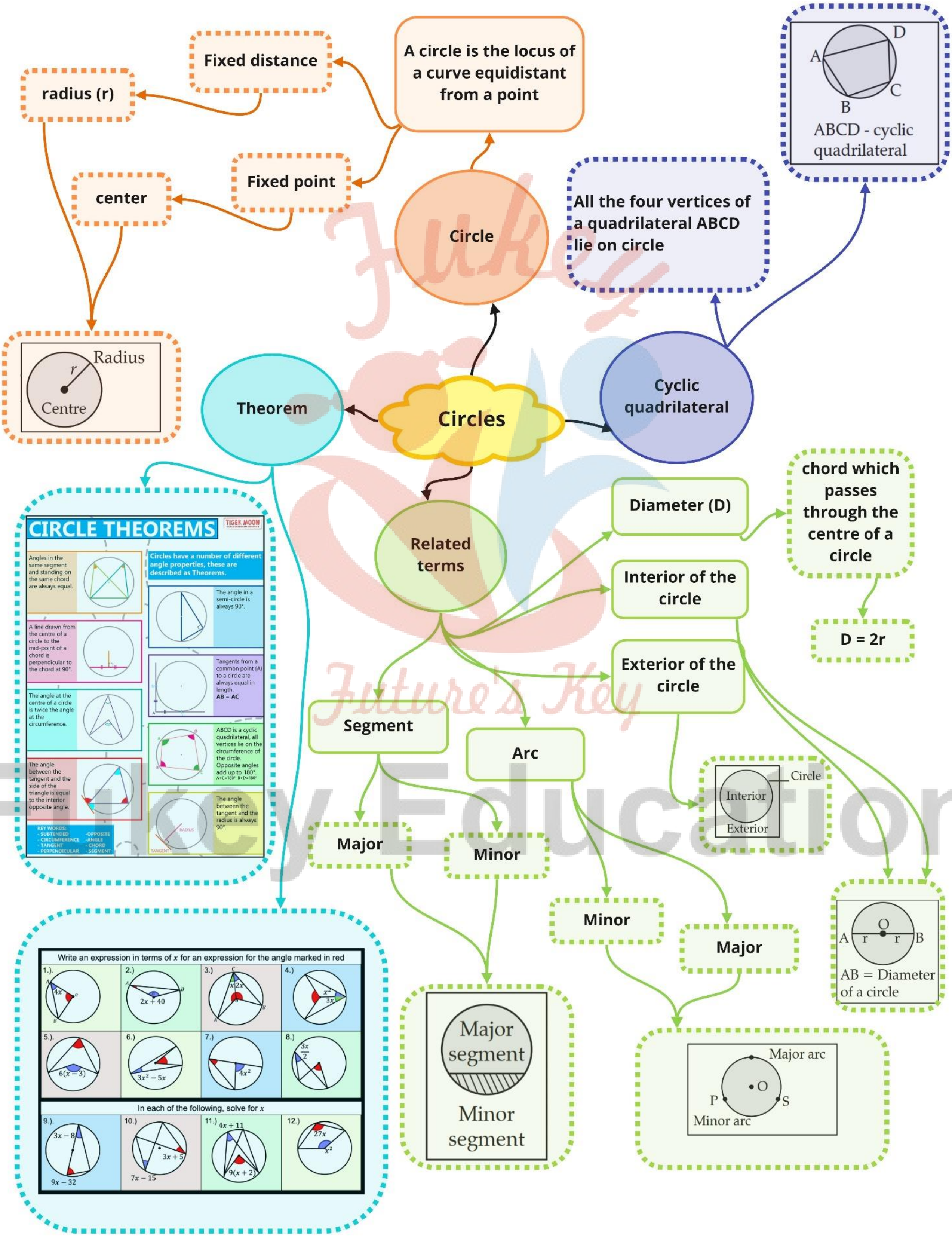
Therefore, ΔOMA ≅ ΔOMB (SSS rule)

⇒ ∠OMA = ∠OMB (C.P.C.T)

But, ∠OMA + ∠OMB = 180^0

Hence, ∠OMA = ∠OMB = 90^0 ⇒ OM⊥AB

Class : 9th mathematics
Chapter- 10: Circles



Important Questions

Multiple Choice questions-

Question 1. If there are two separate circles drawn apart from each other, then the maximum number of common points they have:

(a) 0

(b) 1

(c) 2

(d) 3

Question 2. D is diameter of a circle and AB is a chord. If AD = 50cm, AB = 48cm, then the distance of AB from the Centre of the circle is

(a) 6cm

(b) 8cm

(c) 5cm

(d) 7cm

Question 3. In a circle with center O and a chord BC, points D and E lie on the same side of BC. Then, if ∠BDC = 80°, then ∠BEC =

(a) 80°

(b) 20°

(c) 160°

(d) 40°

Question 4. The center of the circle lies in______ of the circle.

(a) Interior

(b) Exterior

(c) Circumference

(d) None of the above

Question 5. If chords AB and CD of congruent circles subtend equal angles at their centers, then:

(a) AB = CD

(b) AB > CD

(c) AB < AD

(d) None of the above

Question 6. Segment of a circle is the region between an arc and ………. of the circle.

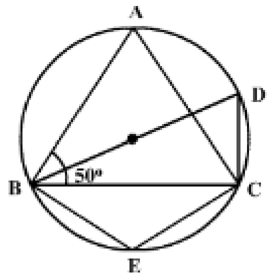
(a) Perpendicular

(b) Radius

(c) Chord

(d) Secant

Question 7. In the figure, triangle ABC is an isosceles triangle with AB = AC and measure of angle ABC = 50°. Then the measure of angle BDC and angle BEC will be



(a) 60°, 100°

(b) 80°, 100°

(c) 50°, 100°

(d) 40°, 120°

Question 8. The region between chord and either of the arc is called.

(a) A sector

(b) A semicircle

(c) A segment

(d) A quarter circles

Question 9. The region between an arc and the two radii joining the Centre of the end points of the arc is called a:

(a) Segment

(b) Semi circle

(c) Minor arc

(d) Sector

Question 10. If a line intersects two concentric circles with Centre O at A, B, C and D, then:

(a) AB = CD

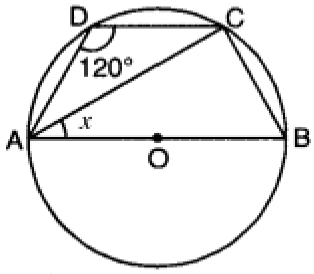
(b) AB > CD

(c) AB < CD

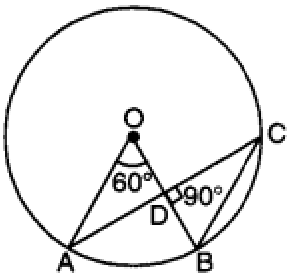
(d) None of the above

Very Short:

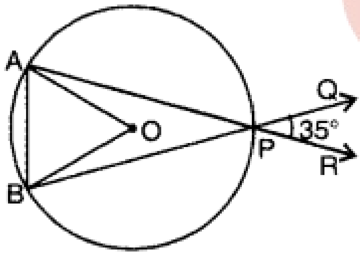
1. In the figure, O is the Centre of a circle passing through points A, B, C and D and ∠ADC = 120°. Find the value of x.



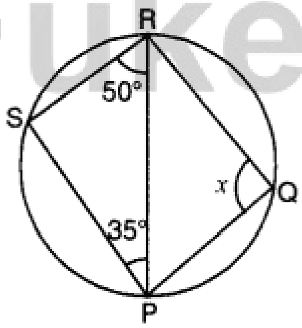
2. In the given figure, O is the Centre of the circle, ∠AOB = 60° and CDB = 90°. Find ∠OBC.



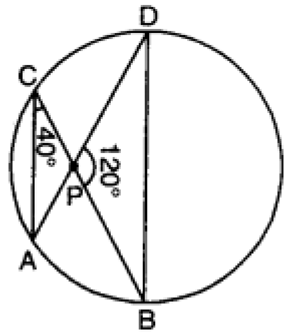
3. In the given figure, O is the Centre of the circle with chords AP and BP being produced to R and Q respectively. If ∠QPR = 35°, find the measure of ∠AOB.



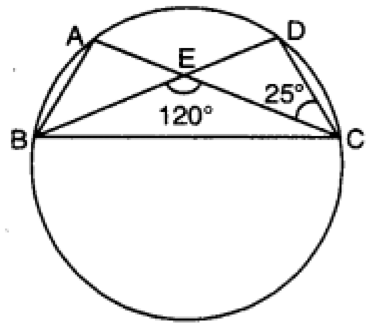
4. In the figure, PQRS is a cyclic quadrilateral. Find the value of x.



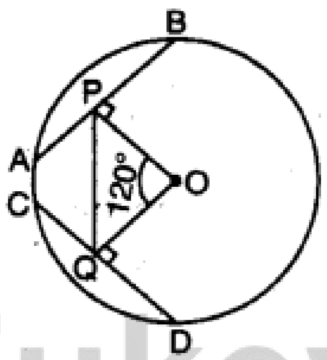
5. In the given figure, ∠ACP = 40° and BPD = 120°, then find ∠CBD.



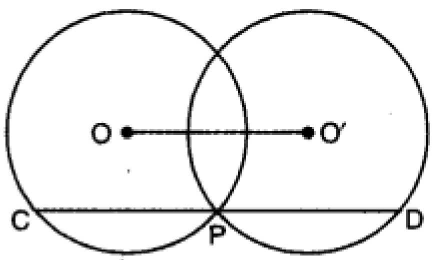
6. In the given figure, if ∠BEC = 120°, ∠DCE = 25°, then find ∠BAC.



7. In the given figure, AB and CD are two equal chords of a circle with Centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠POQ = 120°, find ∠ APQ.

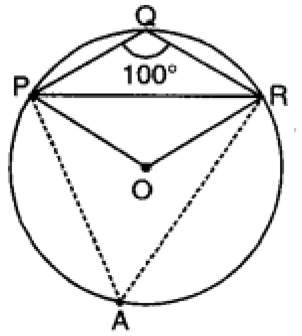


8. Two circles whose centers are O and O' intersect at P. Through P, a line parallel to OO', intersecting the circles at C and D is drawn as shown in the figure. Prove that CD = 2OO'

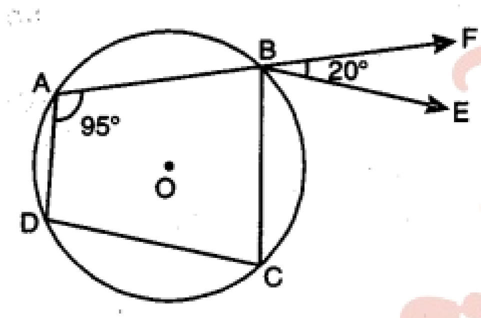


Short Questions:

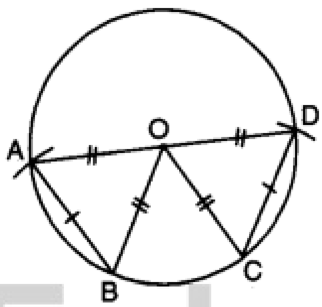
1. In the given figure, PQR = 100°, where P, Q and R are points on a circle with Centre O. Find LOPR.



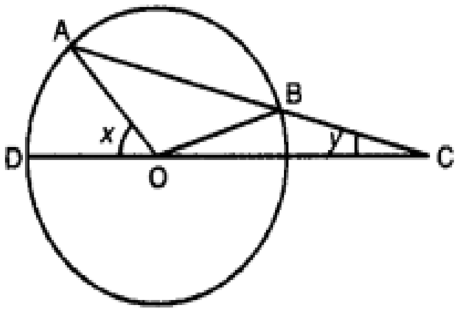
2. In figure, ABCD is a cyclic quadrilateral in which AB is extended to F and BE || DC. If ∠FBE = 20° and DAB = 95°, then find ∠ADC.



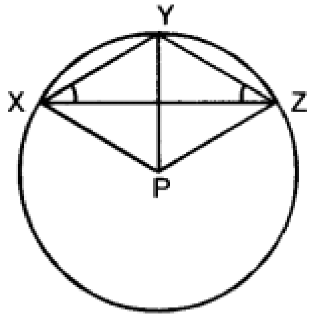
3. If the diagonals of a cyclic quadrilateral are diameters of the circle through the opposite vertices of the quadrilateral. Prove that the quadrilateral is a rectangle.
4. Equal chords of a circle subtends equal angles at the Centre.



5. In the figure, chord AB of circle with Centre O, is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If ∠ACD = y and ∠AOD = x, show that x = 3y.

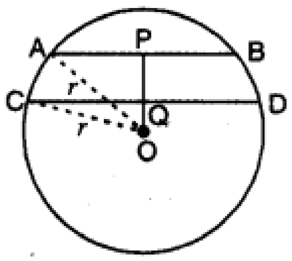


6. In the given figure, P is the Centre of the circle. Prove that: ∠XPZ = 2(∠X∠Y + ∠YXZ).

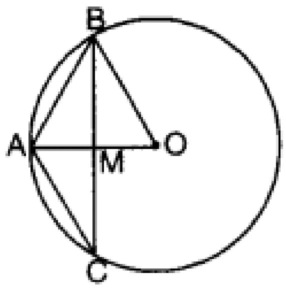


Long Questions:

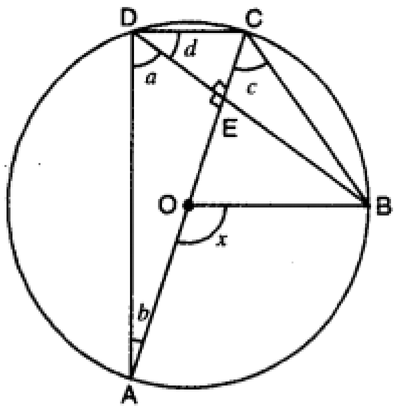
1. In the given figure, O is the Centre of a circle of radius r cm, OP and OQ are perpendiculars to AB and CD respectively and PQ = 1cm. If AB || CD, AB = 6cm and CD = 8cm, determine r



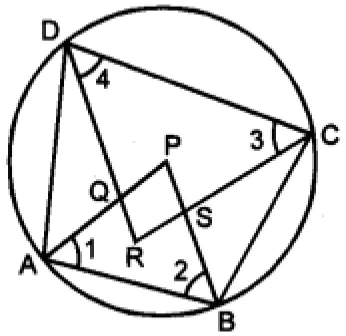
2. In a circle of radius 5cm, AB and AC are two chords such that AB = AC = 6cm, as shown in the figure. Find the length of the chord BC.



3. In the given figure, AC is a diameter of the circle with Centre O. Chord BD is perpendicular to AC. Write down the measures of angles a, b, c and d in terms of x.



4. Show that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



5. PQ and PR are the two chords of a circle of radius r. If the perpendiculars drawn from the Centre of the circle to these chords are of lengths a and b, PQ = 2PR, then

prove that: $b2 = \frac{a2}{4} + \frac{3}{4}r2$

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: In a circle of radius 6 cm, the angle of a sector 60°.Then the area of the sector is 186/7 cm^2.

Reason: Area of the circle with radius r is πr^2.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

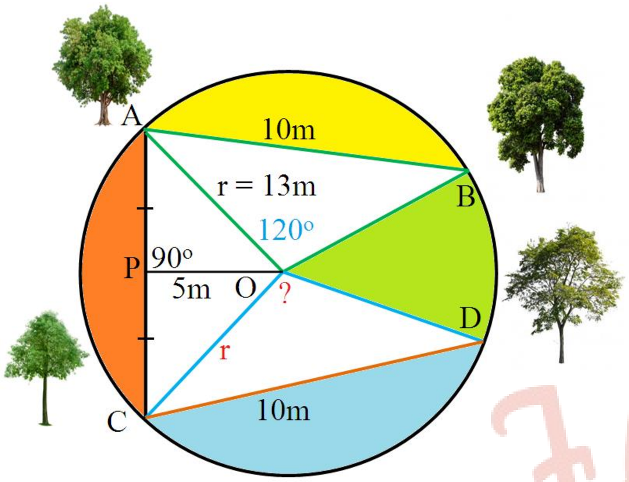
d) Assertion is wrong statement but reason is correct statement.

Assertion: The length of the minute hand of a clock is 7cm. Then the area swept by the minute hand in 5 minutes is $12^5/6cm^2$.

Reason: The length of an arc of a sector of angle θ and radius r is given by l = θ/360 x 2πr

Case Study Questions-

1. Read the Source/ Text given below and answer these questions:



A farmer has a circular garden as shown in the picture above. He has a different type of trees, plants and flower plants in his garden. In the garden, there are two mango trees A and B at a distance of AB = 10m. Similarly, he has two Ashoka trees at the same distance of 10m as shown at C and D. AB subtends ∠AOB=120° at the center O, The perpendicular distance of AC from center is 5m. The radius of the circle is 13m.

Now answer the following questions:

i. What is the value of ∠COD?

a. 60°

b. 120°

c. 100°

d. 80°

ii. What is the distance between mango tree A and Ashok tree C?

a. 12m

b. 24m

c. 13m

d. 15m

iii. What is the value of ∠OAB?

a. 60°

b. 120°

c. 30°

d. 90°

iv. What is the value of ∠OCD?

a. 30°

b. 120°

c. 60°

d. 90°

v. What is the value of ∠ODC?

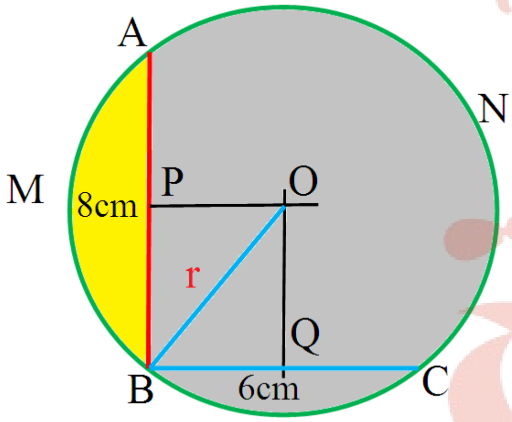
a. 90°

b. 120°

c. 60°

d. 30°

2. Read the Source/ Text given below and answer these questions:



As Class IX C' s teacher Mrs. Rashmi entered in the class, She told students to do some practice on circle chapter. She Draws two-line AB and BC so that AB = 8cm and BC = 6cm. She told all students To make this shape in their notebook and draw a circle passing through the three points A, B and C.

i. Dileep drew AB and BC as per the figure

ii. He drew perpendicular bisectors OP and OQ of the line AB and BC.

iii. OP and OQ intersect at O

iv. Now taking O as centre and OB as radius he drew The circle which passes through A, B and C.

v. He noticed that A, O and C are collinear.

Answer the following questions:

i. What you will call the line AOC?

a. Arc

b. Diameter

c. Radius

d. Chord

ii. What is the measure of ∠ABC?

a. 60°

b. 90°

c. 45°

d. 75°

iii. What you will call the yellow color shaded area AMB?

a. Arc.

b. Sector.

c. Major segment.

d. Minor Segment.

iv. What you will call the grey colour shaded area BCNA?

a. Arc.

b. Sector.

c. Major segment.

d. Minor Segment.

v. What is the radius of the circle?

a. 4cm

b. 3cm

c. 7cm

d. 5cm

Answer Key:

MCQ:

1. (a) 0
2. (d) 7cm
3. (a) 80°
4. (a) Interior
5. (a) AB = CD
6. (c) Chord
7. (b) 80°, 100°
8. (c) A segment
9. (d) Sector
10. (a) AB = CD

Very Short Answer:

1. Since ABCD is a cyclic quadrilateral

   ∠ADC + ∠ABC = 180°

   [∴ opp. ∠s of a cyclic quad. are supplementary]

   120° + ∠ABC = 180°

   ∠ABC = 180° – 120° = 60°

   Now, ∠ACB = 90° [angle in a semicircle]

   In rt. ∠ed ΔCB, ∠ACB = 90°

   ∠CAB + ∠ABC = 90°

   x + 60° = 90°

   x = 90° -60°

   x = 30°

2. Since angle subtended at the Centre by an arc is double the angle

   subtended at the remaining part of the circle.

   ∴ ∠ACB = $\frac{1}{3}$ ∠AOB = $\frac{1}{3}$ x 60° = 30°

   Now, in ACBD, by using angle sum property, we have

   ∠CBD + ∠BDC + ∠DCB = 180°

   ∠CBO + 90° + ∠ACB = 180°

   [∵ ∠CBO = ∠CBD and ∠ACB = ∠DCB are the same ∠s]

   ∠CBO + 90° + 30° = 180°

   ∠CBO = 180o – 90° – 30° = 60°

   or ∠OBC = 60°

3. ∠APB = ∠RPQ = 35° [vert. opp. ∠s]

   Now, ∠AOB and ∠APB are angles subtended by an arc AB at Centre and at the remaining part of the circle.

   ∴ ∠AOB = 2∠APB = 2 × 35° = 70°

4. In ΔPRS, by using angle sum property, we have

∠PSR + ∠SRP + ∠RPS = 180°

∠PSR + 50° + 35o = 180°

∠PSR = 180° – 85o = 95°

Since PQRS is a cyclic quadrilateral

∴ ∠PSR + ∠PQR = 180°

[∵ opp. ∠s of a cyclic quad. are supplementary]

$95° + x = 180°$

$x = 180° – 95°$

$x = 85°$

5. ∠BDP = ∠ACP = 40° [angle in same segment]

Now, in ΔBPD, we have

∠PBD + ∠BPD + ∠BDP = 180°

⇒ ∠PBD + 120° + 40° = 180°

⇒ ∠PBD = 180° – 160o = 20°

or ∠CBD = 20°

6. ∠BEC is exterior angle of ΔCDE.

   ∴ ∠CDE + ∠DCE = ∠BEC

   ⇒ ∠CDE + 25° = 120°

   ⇒ ∠CDE = 95°

7. Arc XY subtends ∠XPY at the Centre P and ∠XZY in the remaining part of the circle.

   ∴ ∠XPY = 2 (∠X∠Y)

   Similarly, arc YZ subtends ∠YPZ at the Centre P and ∠YXZ in the remaining part of the circle.

   ∴ ∠YPZ = 2(∠YXZ) ....(ii)

   Adding (i) and (ii), we have

   ∠XPY + ∠YPZ = 2 (∠XZY + ∠YXZ)

   ∠XP2 = 2 (∠XZY + ∠YXZ)

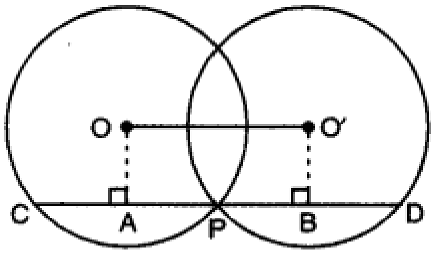
8. Draw OA ⊥ CD and O'B ⊥ CD

   Now, OA ⊥ CD

   OA ⊥ CP

   $CA = AP = \frac{1}{2} CP$

   CP = 2AP ....(i)



Similarly, O'B ⊥ CD

O'B ⊥ PD

⇒ PB = BD = $\frac{1}{2}$ PD

⇒ PD = 2PB

Also, CD = CP + PD

= 2AP + 2PB = 2(AP + PB) = 2AB

CD = 2OO' [∵ OABO' is a rectangle]

Short Answer:

Ans: 1. Take any point A on the circumcircle of the circle.

Join AP and AR.

∵ APQR is a cyclic quadrilateral.

∴ ∠PAR + ∠PQR = 180° [sum of opposite angles of a cyclic quad. is 180°]

∠PAR + 100° = 180°

⇒ Since ∠POR and ∠PAR are the angles subtended by an arc PR at the Centre of the circle and circumcircle of the circle.

∠POR = 2∠PAR = 2 x 80° = 160°

∴ In APOR, we have OP = OR [radii of same circle]

∠OPR = ∠ORP [angles opposite to equal sides]

Now, ∠POR + ∠OPR + ∠ORP = 180°

⇒ 160° + ∠OPR + ∠OPR = 180°

⇒ 2∠OPR = 20°

⇒ ∠OPR = 10°

Ans: 2. Sum of opposite angles of a cyclic quadrilateral is 180°

∴ ∠DAB + ∠BCD = 180°

⇒ 95° + ∠BCD = 180°

⇒ ∠BCD = 180° – 95° = 85°

∵ BE || DC

∴ ∠CBE = ∠BCD = 85°[alternate interior angles]

∴ ∠CBF = CBE + ∠FBE = 85° + 20° = 105°

Now, ∠ABC + 2CBF = 180° [linear pair]

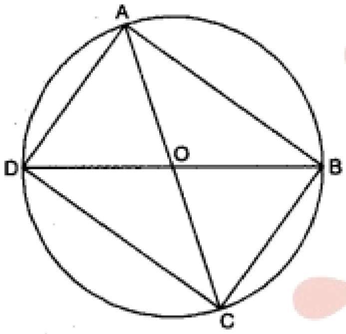
and ∠ABC + ∠ADC = 180° [opposite angles of cyclic quad.]

Thus, ∠ABC + ∠ADC = ∠ABC + 2CBF

⇒ ∠ADC = CBF

⇒ ∠ADC = 105° [∵ CBF = 105°]

Ans: 3. Here, ABCD is a cyclic quadrilateral in which AC and BD are diameters.



Since AC is a diameter.

∴ ∠ABC = ∠ADC = 90°

[∵ angle of a semicircle = 90°]

Also, BD is a diameter

∴ ∠BAD = ∠BCD = 90° [∵ angle of a semicircle = 90°]

Now, all the angles of a cyclic quadrilateral ABCD are 90 each.

Hence, ABCD is a rectangle.

Ans: 4. Given: In a circle C(O, r), chord AB = chord CD

To Prove: ∠AOB = ∠COD.

Proof: In ΔAOB and ΔCOD

AO = CO (radii of same circle]

BO = DO [radii of same circle]

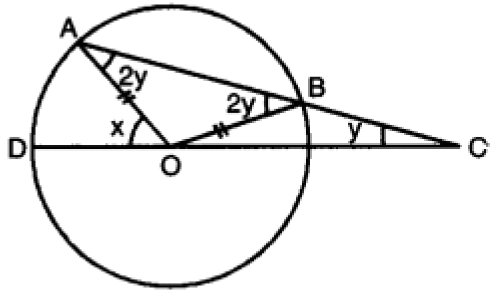
Chord AB = Chord CD (given]

⇒ ΔAOB = ACOD [by SSS congruence axiom]

⇒ ∠AOB = COD (c.p.c.t.]

Ans: 5. In AOBC, OB = BC

⇒ ∠BOC = ∠BCO = y [angles opp. to equal sides are equal]



∠OBA is the exterior angle of ΔBOC

So, ∠ABO = 2y [ext. angle is equal to the sum of int. opp. angles]

Similarly, ∠AOD is the exterior angle of ΔAOC

∴ x = 2y + y = 3y

Ans: 6. Arc XY subtends ∠XPY at the Centre P and ∠XZY in the remaining part of the circle.

∴ ∠XPY = 2 (∠X∠Y)

Similarly, arc YZ subtends ∠YPZ at the Centre P and ∠YXZ in the remaining part of the circle.

∴ ∠YPZ = 2(∠YXZ) ….(ii)

Adding (i) and (ii), we have

∠XPY + ∠YPZ = 2 (∠XZY + ∠YXZ)

∠XP2 = 2 (∠XZY + ∠YXZ)

Long Answer:

Ans: 1. Since the perpendicular drawn from the Centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

Consequently, AP = BP = $\frac{1}{2}$ AB = 3cm

and CQ = QD = $\frac{1}{2}$ CD = 4cm

In right-angled AQAP, we have

$OA^2 = OP^2 + AP2$

$r^2 = OP^2 + 32$

$r^2 = OP^2 + 9$

In right-angled ΔOCQ, we have

$OC^2 = OQ^2 + CQ^2$

$r^2 = OQ^2 + 42$

$p^2 = OQ^2 + 16$ … (ii)

From (i) and (ii), we have

$OP^2 + 9 = OQ^2 + 16$

$OP^2 - OQ^2 = 16 - 9$

$x^2 - (x - 1)^2 = 16 - 9$ [where OP = x and PQ = 1cm given]

$x^2 - y^2 - 1 + 2x = 7$

$2x = 7 + 1$

$x = 4$

⇒ OP = 4cm

From (i), we have

$r^2 = (4)^2 + 9$

$r^2 = 16 + 9 = 25$

r = 5cm

Ans: 2. Here, OA = OB = 5cm [radii]

AB = AC = 6cm

∴ B and C are equidistant from A.

∴ AO is the perpendicular bisector of chord BC and it intersect BC in M.

Now, in rt. ∠ed ΔAMB, M = 90° .... (i)

∴ By using Pythagoras Theorem, we have

$BM^2 = AB^2 - AM^2$

$= 36 - AM^2$

Also, in rt. ∠ed ΔBMO, ∠M = 90°

∴ By using Pythagoras Theorem, we have

$BM^2 = BO^2 - MO^2 = 25 - (AO - AM)^2$

From (i) and (ii), we obtain

$25 - (AO - AM)^2 = 36 - AM12$

$25 - AOC - AM^2 + 240 \times AM = 36 - AM12$

$25 - 25 + 2 \times 5 \times AM = 36$

10 AM = 36

AM = 3.6cm

From (i), we have

$BM^2 = 36 - (3.6)2 = 36 - 12.96 = 23.04$

BM = √23.04 = 4.8cm

Thus, BC = 2 × BM

= 2 × 4.8 = 9.6cm

Hence, the length of the chord BC is 9.6cm.

Ans: 3. Here, AC is a diameter of the circle.

∴ ∠ADC = 90°

⇒ ∠a + ∠d = 90°

In right-angled ΔAED, ∠E = 90°

∴ ∠a + 2b = 90°

From (i) and (ii), we obtain

∠b = ∠d …. (iii)

Also, ∠a = ∠c … (iv)

[∠s subtended by the same segment are equal]

Now, ∠AOB and ∠ADB are angles subtended by an arc AB at the Centre and at the remaining part of the circle.

$$\therefore \angle ADB = \frac{1}{2}\angle AOB \Rightarrow \angle a = \frac{x}{2}$$

From (*iv*), we have $\angle a = \angle c = \frac{x}{2}$

Again, $\angle AOB + \angle BOC = 180°$

$$\Rightarrow \angle BOC = 180° - \angle AOB = 180° - x$$

∠BOC and ∠BDC are angles subtended by an arc BC at the centre and at the remaining part of the circle.

$$\therefore \angle BDC = \frac{1}{2}\angle BOC$$

$$\Rightarrow \angle d = \frac{1}{2}(180° - x) = 90° - \frac{x}{2}$$

Ans: 4. Given: A cyclic quadrilateral ABCD in which AP, BP, CR and DR are the angle bisectors of ∠A, ∠B, 2C and ∠D respectively such that a quadrilateral PQRS is formed. To

Prove: PQRS is a cyclic quadrilateral.

Proof: Since ABCD is a cyclic quadrilateral.

∴ ∠A + 2C = 180° and ∠B + ∠D = 180°

Also, AP, BP, CR and DR are the angle bisectors of ∠A, ∠B, ∠C and ∠D respectively.

$\therefore\ \angle 1 = \frac{1}{2}\angle A,\ \angle 2 = \frac{1}{2}\angle B,\ \angle 3 = \frac{1}{2}\angle C$ and $\angle 4 = \frac{1}{2}\angle D$

From (*i*), we have $\frac{1}{2}\angle A + \frac{1}{2}\angle C = \frac{1}{2}(\angle A + \angle C) = \frac{1}{2} \times 180^\circ = 90^\circ$

and $\frac{1}{2}\angle B + \frac{1}{2}\angle D = \frac{1}{2}(\angle B + \angle D) = \frac{1}{2} \times 180^\circ = 90^\circ$

or ∠1 + ∠3 = 90°

and ∠2 + ∠4 = 90°

Now, in ΔAPB, by angle sum property of a Δ

∠1 + ∠2 + ∠P = 180° … (iii)

Again, in ΔCRD, by angle sum property of a Δ

∠3 + ∠4 + ∠R = 180° …(iv)

Adding (iii) and (iv), we have

∠1 + ∠2 + ∠3 + ∠4 + ∠P + ∠R = 180° + 180°

90° + 90° + ∠P + ∠R = 360° [using (ii)]

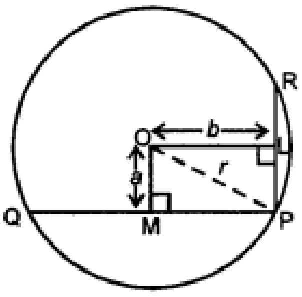
∠P + ∠R= 360° – 180° = 180°

i.e., the sum of one pair of the opposite angles of quadrilateral PQRS is 180°.

Hence, the quadrilateral PQRS is a cyclic quadrilateral.

Ans: 5. In circle Clo, r), PQ and PR are two chords, draw OM I PQ, OL I PR, such that OM = a

and OL = b. Join OP. Since the perpendicular from the Centre of the circle to the chord of the circle, bisects the chord.



∴ We have, $PM = MQ = \frac{1}{2}PQ$

and $PL = LR = \frac{1}{2}PR$

In rt. ΔOMP, ∠M = 90°

∴ By Pythagoras Theorem, we have

$$PM^2 = OP^2 - OM^2$$

$$\left(\frac{1}{2}PQ\right)^2 = r^2 - a^2$$

$$\frac{PQ^2}{4} = r^2 - a^2$$

$$\Rightarrow \quad PQ^2 = 4r^2 - 4a^2 \quad ...(i)$$

Again, in rt. ΔOLP, ∠L = 90°

∴ By Pythagoras Theorem, we have

$$PL^2 = OP^2 - OL^2$$

$$\left(\frac{1}{2}PR\right)^2 = r^2 - b^2$$

$$\frac{PR^2}{4} = r^2 - b^2$$

$$PR^2 = 4r^2 - 4b^2 \quad ...(ii)$$

Also, $PQ = 2PR$ [given]

$$\Rightarrow \quad PQ^2 = 4PR^2 \quad ...(iii)$$

From (i), (ii) and (iii), we have

$$4r^2 - 4a^2 = 4(4r^2 - 4b^2)$$

$$\Rightarrow \quad r^2 - a^2 = 4r^2 - 4b^2$$

$$\Rightarrow \quad 4b^2 = 4r^2 - r^2 + a^2$$

$$\Rightarrow \quad 4b^2 = 3r^2 + a^2$$

$$\Rightarrow \quad b^2 = \frac{3}{4}r^2 + \frac{1}{4}a^2$$

or $$b^2 = \frac{1}{4}a^2 + \frac{3}{4}r^2$$

Assertion and Reason Answers-

1. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

2. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Case Study Answers-

1.

(i)	(b)	120°
(ii)	(b)	24m
(iii)	(c)	30°
(iv)	(a)	30°

(v)	(d)	30°

2.

(i)	(b)	Diameter
(ii)	(b)	90°
(iii)	(d)	Minor Segment.
(iv)	(c)	Major segment.
(v)	(d)	5cm