

MATHEMATICS

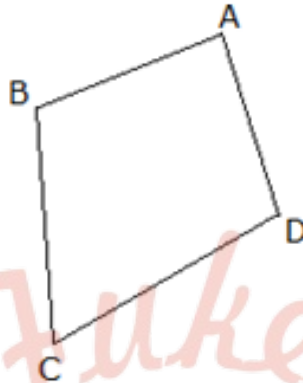
Chapter 8: Quadrilaterals



Quadrilaterals

Quadrilateral

A **quadrilateral** is a closed figure obtained by joining four points (with no three points collinear) in an order.



Here, ABCD is a quadrilateral.

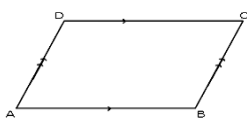
Parts of a quadrilateral

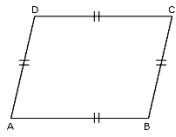
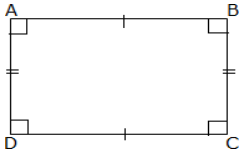
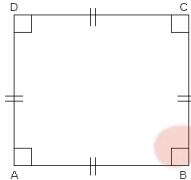
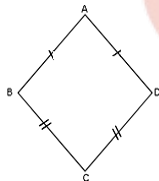
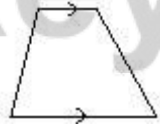
- A quadrilateral has four sides, four angles and four vertices.
- Two sides of a quadrilateral having no common end point are called its **opposite sides**.
- Two sides of a quadrilateral having a common end point are called its **adjacent sides**.
- Two angles of a quadrilateral having common arm are called its **adjacent angles**.
- Two angles of a quadrilateral not having a common arm are called its **opposite angles**.
- A **diagonal** is a line segment obtained on joining the opposite vertices.

Angle sum property of a quadrilateral

Sum of all the angles of a quadrilateral is 360° . This is known as the **angle sum property of a quadrilateral**.

Types of quadrilaterals and their properties:

Name of a quadrilateral	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	i. Opposite sides are equal. ii. Opposite angles are equal. iii. Diagonals bisect one another.

<p>Rhombus: A parallelogram with sides of equal length.</p> 	<ol style="list-style-type: none"> All properties of a parallelogram. Diagonals are perpendicular to each other.
<p>Rectangle: A parallelogram with all angles right angle.</p> 	<ol style="list-style-type: none"> All the properties of a parallelogram. Each of the angles is a right angle. Diagonals are equal.
<p>Square: A rectangle with sides of equal length.</p> 	<p>All the properties of a parallelogram, a rhombus and a rectangle.</p>
<p>Kite: A quadrilateral with exactly two pairs of equal consecutive sides.</p> 	<ol style="list-style-type: none"> The diagonals are perpendicular to one another. One of the diagonals bisects the other. If ABCD is a kite, then $\angle B = \angle D$ but $\angle A \neq \angle C$
<p>Trapezium: A quadrilateral with one pair of opposite sides parallel is called trapezium.</p> 	<p>One pair of opposite sides parallel.</p>

Important facts about quadrilaterals

- If the non-parallel sides of trapezium are equal, it is known as **isosceles trapezium**.
- Square, rectangle and rhombus are all parallelograms.
- Kite and trapezium are not parallelograms.
- A square is a rectangle.
- A square is a rhombus.
- A parallelogram is a trapezium.

A quadrilateral is a parallelogram if:

- i. each pair of opposite sides of a quadrilateral is equal, or
- ii. each pair of opposite angles is equal, or
- iii. the diagonals of a quadrilateral bisect each other, or
- iv. each pair of opposite sides is equal and parallel.

Mid-Point Theorem

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Converse of mid-point theorem

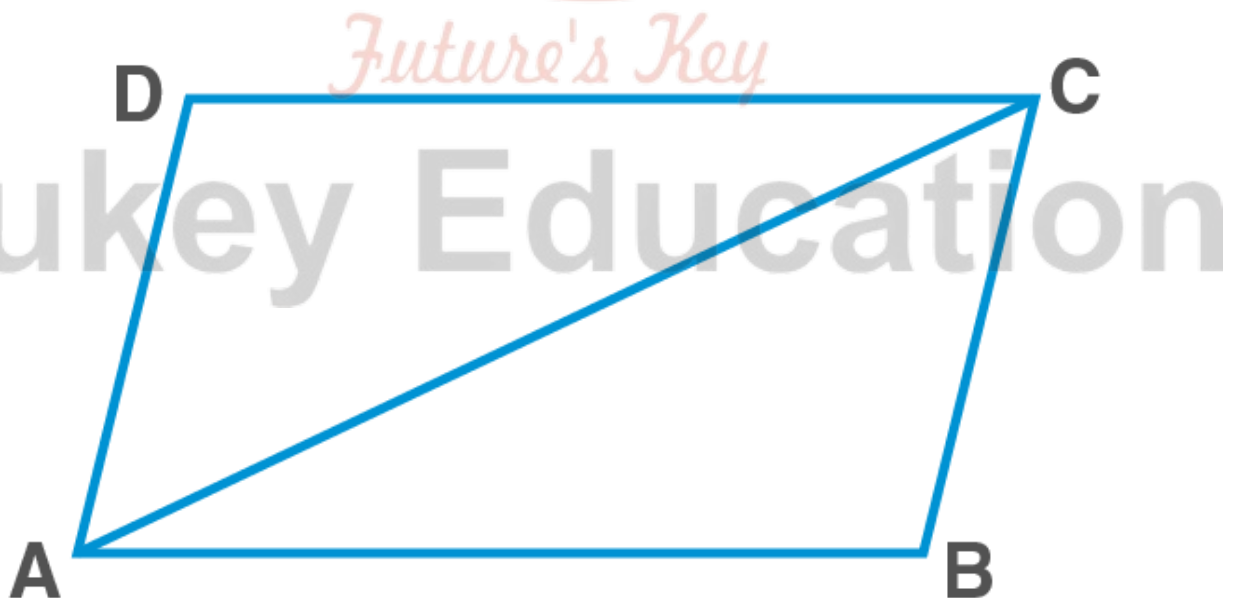
The line drawn through the mid-point of one side of a triangle, parallel to another side, bisects the third side.

Formation of a new quadrilateral using the given data

- If the diagonals of a parallelogram are equal, then it is a rectangle.
- If the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.
- If the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

Parallelogram: Opposite sides of a parallelogram are equal



In $\triangle ABC$ and $\triangle CDA$

$AC = AC$ [Common / transversal]

$$\angle BCA = \angle DAC \text{ [alternate angles]}$$

$$\angle BAC = \angle DCA \text{ [alternate angles]}$$

$$\triangle ABC \cong \triangle CDA \text{ [ASA rule]}$$

Hence,

$$AB = DC \text{ and } AD = BC \text{ [C.P.C.T.C]}$$

Opposite angles in a parallelogram are equal



In parallelogram ABCD

$AB \parallel CD$; and AC is the transversal

Hence, $\angle 1 = \angle 3$ (1) (alternate interior angles)

$BC \parallel DA$; and AC is the transversal

Hence, $\angle 2 = \angle 4$ (2) (alternate interior angles)

Adding (1) and (2)

$$\angle 1 + \angle 2 = \angle 3 + \angle 4$$

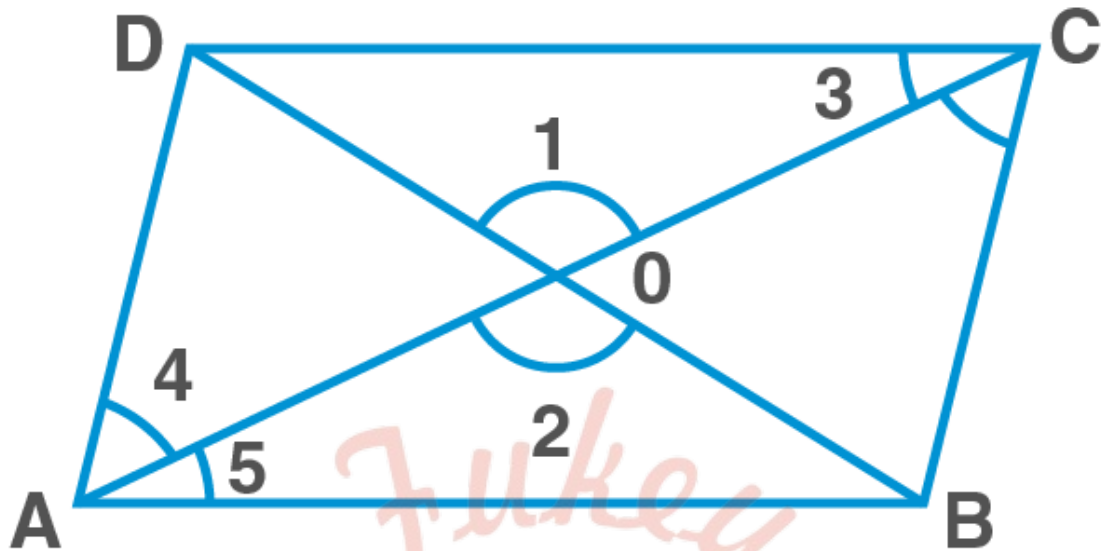
$$\angle BAD = \angle BCD$$

Similarly,

$$\angle ADC = \angle ABC$$

Properties of diagonal of a parallelogram

Diagonals of a parallelogram bisect each other.



In $\triangle AOB$ and $\triangle COD$,

$\angle 3 = \angle 5$ [alternate interior angles]

$\angle 1 = \angle 2$ [vertically opposite angles]

$AB = CD$ [opp. Sides of parallelogram]

$\triangle AOB \cong \triangle COD$ [AAS rule]

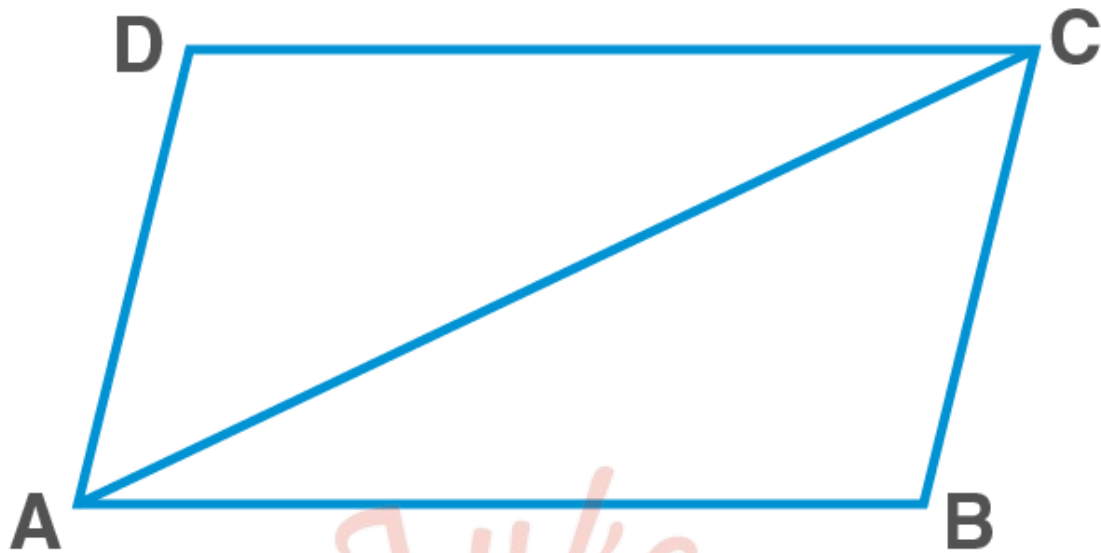
$OB = OD$ and $OA = OC$ [C.P.C.T]

Hence, proved

Conversely,

If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Diagonal of a parallelogram divides it into two congruent triangles.



In $\triangle ABC$ and $\triangle CDA$,

$AB = CD$ [Opposite sides of parallelogram]

$BC = AD$ [Opposite sides of parallelogram]

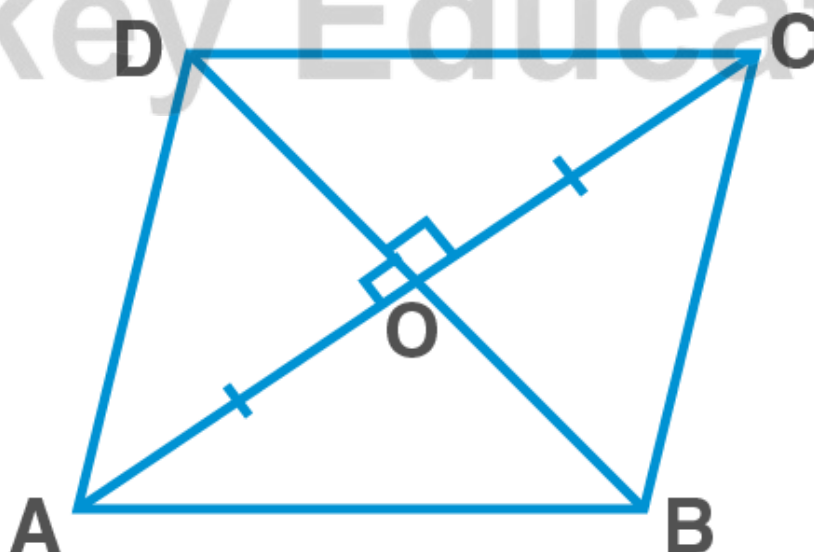
$AC = AC$ [Common side]

$\triangle ABC \cong \triangle CDA$ [by SSS rule]

Hence, proved.

Diagonals of a rhombus bisect each other at right angles

Diagonals of a rhombus bisect each – other at right angles



In $\triangle AOD$ and $\triangle COD$,

$OA = OC$ [Diagonals of parallelogram bisect each other]

$OD = OD$ [Common side]

$AD = CD$ [Adjacent sides of a rhombus]

$\triangle AOD \cong \triangle COD$ [SSS rule]

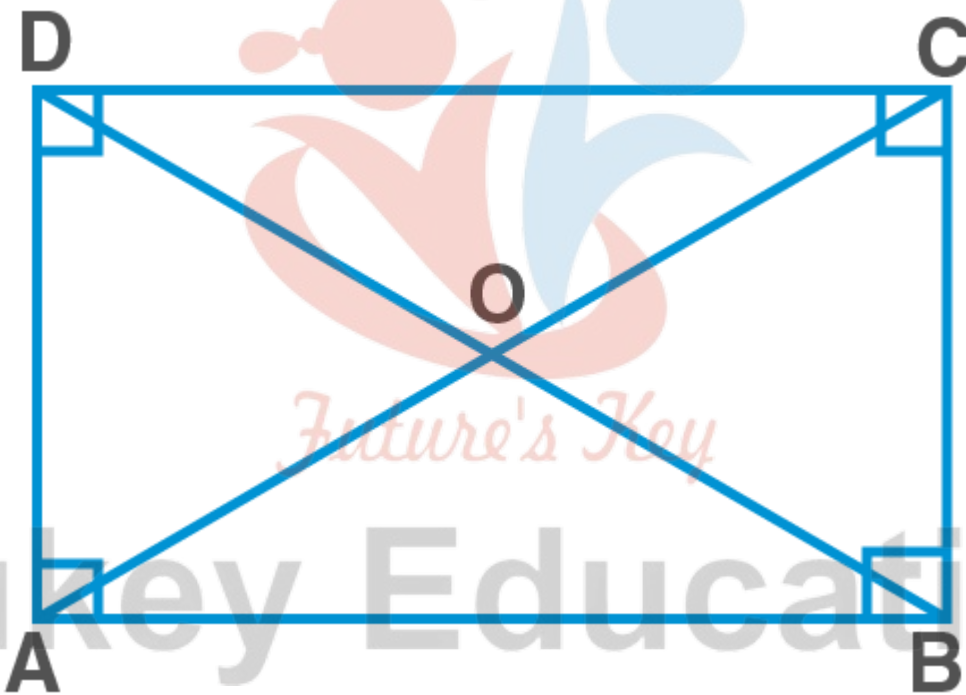
$\angle AOD = \angle DOC$ [C.P.C.T]

$\angle AOD + \angle DOC = 180$ [\because AOC is a straight line]

Hence, $\angle AOD = \angle DOC = 90$

Hence proved.

Diagonals of a rectangle bisect each other and are equal



Rectangle ABCD

In $\triangle ABC$ and $\triangle BAD$,

$AB = BA$ [Common side]

$BC = AD$ [Opposite sides of a rectangle]

$\angle ABC = \angle BAD$ [Each = $90^\circ \because$ ABCD is a Rectangle]

$\triangle ABC \cong \triangle BAD$ [SAS rule]

$\therefore AC = BD$ [C.P.C.T]

Consider $\triangle OAD$ and $\triangle OCB$,

$AD = CB$ [Opposite sides of a rectangle]

$\angle OAD = \angle OCB$ [$\because AD \parallel BC$ and transversal AC intersects them]

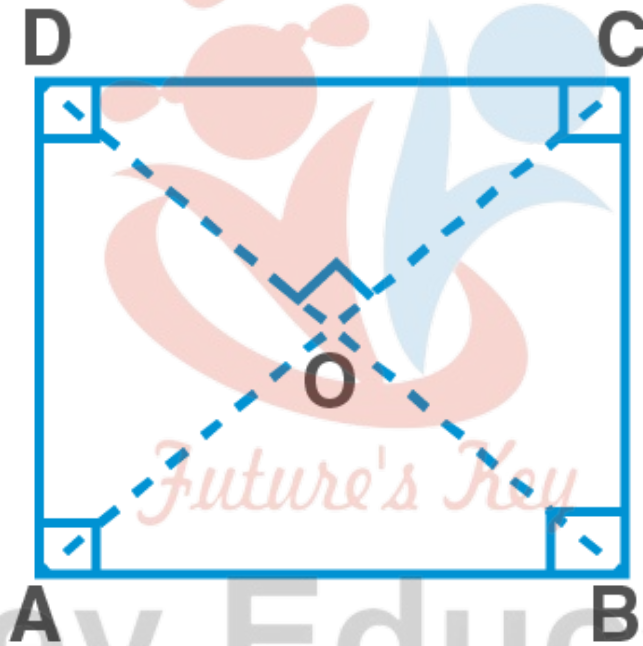
$\angle ODA = \angle OBC$ [$\because AD \parallel BC$ and transversal BD intersects them]

$\triangle OAD \cong \triangle OCB$ [ASA rule]

$\therefore OA = OC$ [C.P.C.T]

Similarly, we can prove $OB = OD$

Diagonals of a square bisect each other at right angles and are equal



In $\triangle ABC$ and $\triangle BAD$,

$AB = BA$ [Common side]

$BC = AD$ [Opposite sides of a Square]

$\angle ABC = \angle BAD$ [Each = $90^\circ \because ABCD$ is a Square]

$\triangle ABC \cong \triangle BAD$ [SAS rule]

$\therefore AC = BD$ [C.P.C.T]

Consider $\triangle OAD$ and $\triangle OCB$,

$AD = CB$ [Opposite sides of a Square]

$\angle OAD = \angle OCB$ [$\because AD \parallel BC$ and transversal AC intersects them]

$\angle ODA = \angle OBC$ [$\because AD \parallel BC$ and transversal BD intersects them]

$\triangle OAD \cong \triangle OCB$ [ASA rule]

$\therefore OA = OC$ [C.P.C.T]

Similarly, we can prove $OB = OD$

In $\triangle OBA$ and $\triangle ODA$,

$OB = OD$ [proved above]

$BA = DA$ [Sides of a Square]

$OA = OA$ [Common side]

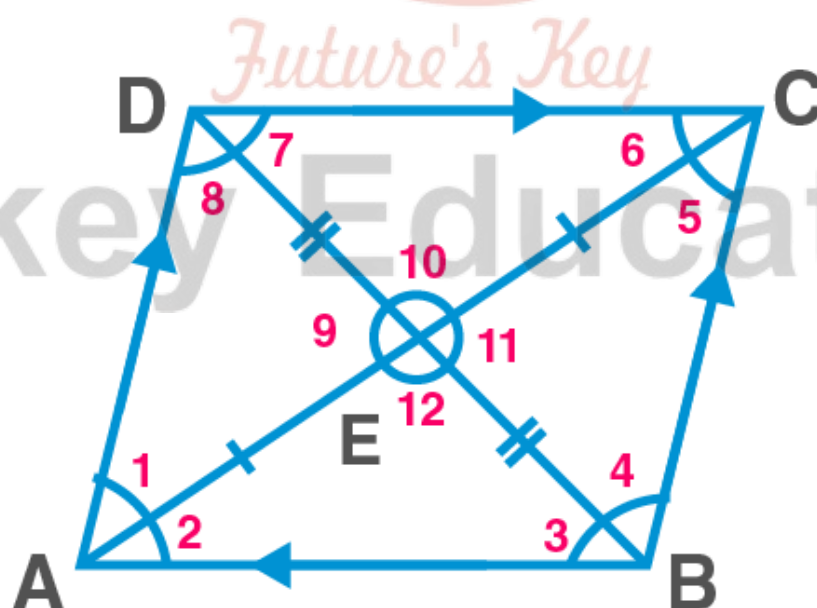
$\triangle OBA \cong \triangle ODA$, [SSS rule]

$\therefore \angle AOB = \angle AOD$ [C.P.C.T]

But $\angle AOB + \angle AOD = 180^\circ$ [Linear pair]

$\therefore \angle AOB = \angle AOD = 90^\circ$

Important results related to parallelograms



Opposite sides of a parallelogram are parallel and equal.

$AB \parallel CD, AD \parallel BC, AB = CD, AD = BC$

Opposite angles of a parallelogram are equal adjacent angles are supplementary.

$$\angle A = \angle C, \angle B = \angle D,$$

$$\angle A + \angle B = 180^\circ, \angle B + \angle C = 180^\circ, \angle C + \angle D = 180^\circ, \angle D + \angle A = 180^\circ$$

A diagonal of parallelogram divides it into two congruent triangles.

$$\triangle ABC \cong \triangle CDA \text{ [With respect to AC as diagonal]}$$

$$\triangle ADB \cong \triangle CBD \text{ [With respect to BD as diagonal]}$$

The diagonals of a parallelogram bisect each other.

$$AE = CE, BE = DE$$

$$\angle 1 = \angle 5 \text{ (alternate interior angles)}$$

$$\angle 2 = \angle 6 \text{ (alternate interior angles)}$$

$$\angle 3 = \angle 7 \text{ (alternate interior angles)}$$

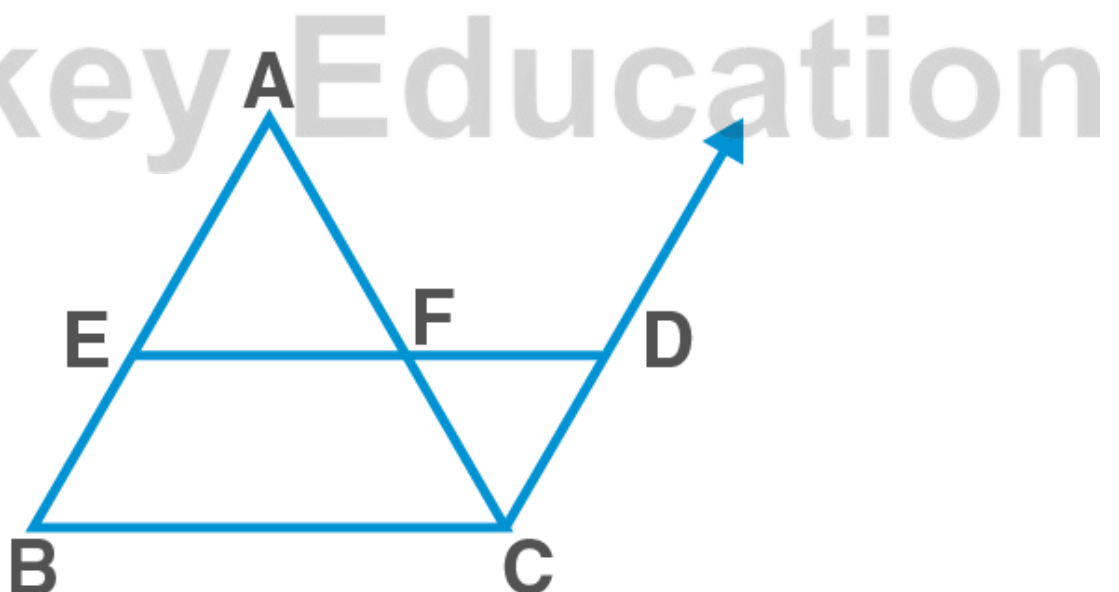
$$\angle 4 = \angle 8 \text{ (alternate interior angles)}$$

$$\angle 9 = \angle 11 \text{ (vertically opp. angles)}$$

$$\angle 10 = \angle 12 \text{ (vertically opp. angles)}$$

The Mid-Point Theorem

The line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half of the third side



In $\triangle ABC$, E – the midpoint of AB; F – the midpoint of AC

Construction: Produce EF to D such that $EF = DF$.

In $\triangle AEF$ and $\triangle CDF$,

$AF = CF$ [F is the midpoint of AC]

$\angle AFE = \angle CFD$ [V.O.A]

$EF = DF$ [Construction]

$\therefore \triangle AEF \cong \triangle CDF$ [SAS rule]

Hence,

$\angle EAF = \angle DCF$ (1)

$DC = EA = EB$ [E is the midpoint of AB]

$DC \parallel EA \parallel AB$ [Since, (1), alternate interior angles]

$DC \parallel EB$

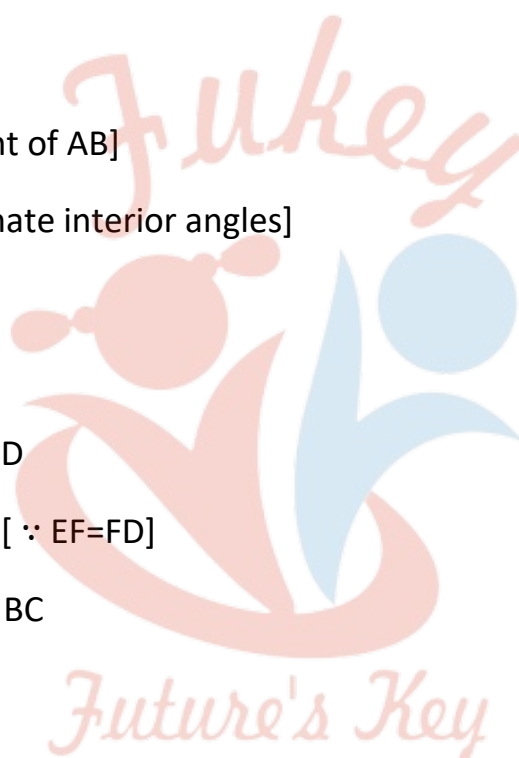
So EBCD is a parallelogram

Therefore, $BC = ED$ and $BC \parallel ED$

Since $ED = EF + FD = 2EF = BC$ [$\because EF=FD$]

We have, $EF = \frac{1}{2} BC$ and $EF \parallel BC$

Hence proved.



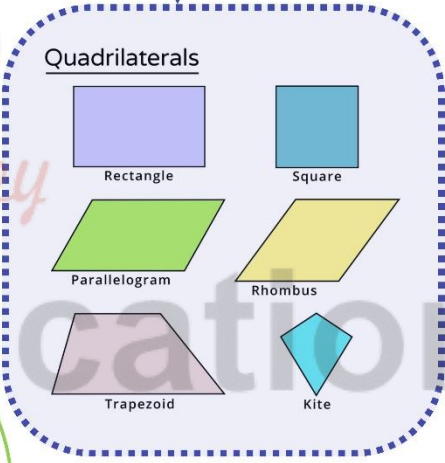
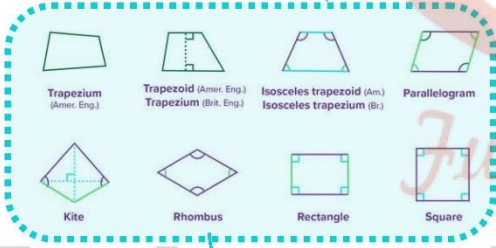
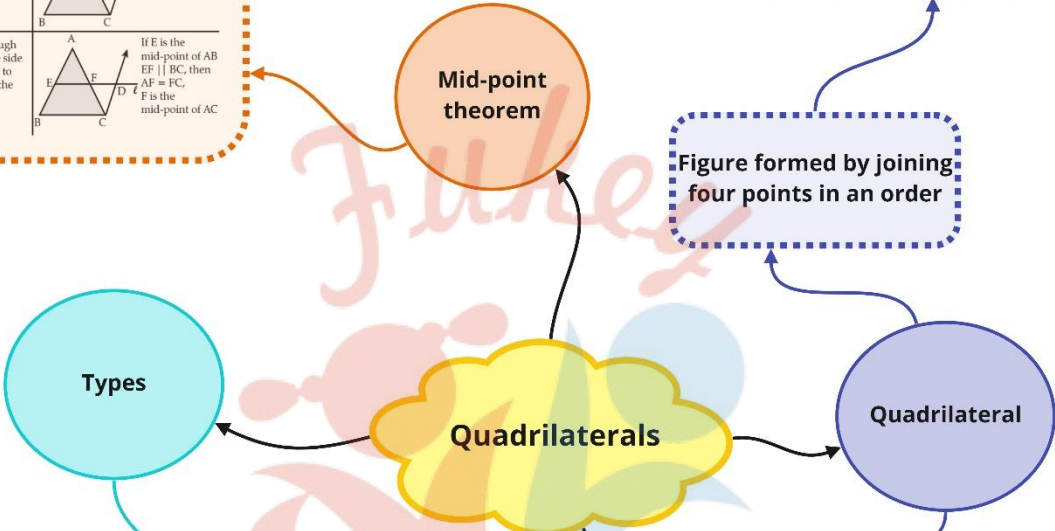
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Class : 9th mathematics
Chapter- 8: Quadrilaterals

Statement	Figure
The line-segment joining the mid-points of two sides of a triangle is parallel to the third side.	
The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side	

It has four - vertices, angles and sides each

Figure formed by joining four points in an order



Sides and angles	Square	Rectangle	Rhombus	Parallelogram	Trapezium
All sides are equal	Yes	No	Yes	No	No
Opposite sides are parallel	Yes	Yes	Yes	Yes	Yes
Opposite sides are equal	Yes	Yes	Yes	Yes	No
All the angles are of the same measure	Yes	Yes	No	No	No
Opposite angles are of equal measure	Yes	Yes	Yes	Yes	No
Diagonals bisect each other	Yes	Yes	Yes	Yes	No
Two adjacent angles are supplementary	Yes	Yes	Yes	Yes	No

Quadrilateral	Properties
Rectangle	4 right angles and opposite sides equal
Square	4 right angles and 4 equal sides
Parallelogram	Two pairs of parallel sides and opposite sides equal
Rhombus	Parallelogram with 4 equal sides
Trapezoid	Two sides are parallel
Kite	Two pairs of adjacent sides of the same length

Important Questions

Multiple Choice questions-

Question 1. A diagonal of a Rectangle is inclined to one side of the rectangle at an angle of 25° . The Acute Angle between the diagonals is:

- (a) 115°
- (b) 50°
- (c) 40°
- (d) 25°

Question 2. The diagonals of a rectangle PQRS intersect at O. If $\angle QOR = 44^\circ$, $\angle OPS = ?$

- (a) 82°
- (b) 52°
- (c) 68°
- (d) 75°

Question 3. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is

- (a) Rhombus
- (b) Parallelogram
- (c) Trapezium
- (d) Kite

Question 4. All the angles of a convex quadrilateral are congruent. However, not all its sides are congruent. What type of quadrilateral is it?

- (a) Parallelogram
- (b) Square
- (c) Rectangle
- (d) Trapezium

Question 5. In a Quadrilateral ABCD, $AB = BC$ and $CD = DA$, then the quadrilateral is a

- (a) Triangle
- (b) Kite
- (c) Rhombus
- (d) Rectangle

Question 6. The angles of a quadrilateral are $(5x)^\circ$, $(3x + 10)^\circ$, $(6x - 20)^\circ$ and $(x$

+ 25)°. Now, the measure of each angle of the quadrilateral will be

- (a) 115°, 79°, 118°, 48°
- (b) 100°, 79°, 118°, 63°
- (c) 110°, 84°, 106°, 60°
- (d) 75°, 89°, 128°, 68°

Question 7. The diagonals of rhombus are 12 cm and 16 cm. The length of the side of rhombus is:

- (a) 12cm
- (b) 16cm
- (c) 8cm
- (d) 10cm

Question 8. In quadrilateral PQRS, if $\angle P = 60^\circ$ and $\angle Q : \angle R : \angle S = 2 : 3 : 7$, then $\angle S =$

- (a) 175°
- (b) 210°
- (c) 150°
- (d) 135°

Question 9. In parallelogram ABCD, if $\angle A = 2x + 15^\circ$, $\angle B = 3x - 25^\circ$, then value of x is:

- (a) 91°
- (b) 89°
- (c) 34°
- (d) 38°

Question 10. If ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$, then:

- (a) $\angle A = \angle B$
- (b) $\angle A > \angle B$
- (c) $\angle A < \angle B$
- (d) None of the above

Very Short:

1. If one angle of a parallelogram is twice of its adjacent angle, find the angles of the parallelogram.
2. If the diagonals of a quadrilateral bisect each other at right angles, then name the quadrilateral.

- Three angles of a quadrilateral are equal, and the fourth angle is equal to 144° . Find each of the equal angles of the quadrilateral.
- If ABCD is a parallelogram, then what is the measure of $\angle A - \angle C$?
- PQRS is a parallelogram, in which $PQ = 12$ cm and its perimeter is 40 cm. Find the length of each side of the parallelogram.
- Two consecutive angles of a parallelogram are $(x + 60)^\circ$ and $(2x + 30)^\circ$. What special name can you give to this parallelogram?
- ONKA is a square with $\angle KON = 45^\circ$. Determine $\angle KOA$.
- In quadrilateral PQRS, if $\angle P = 60^\circ$ and $\angle Q : \angle R : \angle S = 2 : 3 : 7$, then find the measure of $\angle S$.

Short Questions:

- ABCD is a parallelogram in which $\angle ADC = 75^\circ$ and side AB is produced to point E as shown in the figure. Find $x + y$.
- If the diagonals of a parallelogram are equal, then show that it is a rectangle.
- In the figure, ABCD is a rhombus, whose diagonals meet at O. Find the values of x and y .
- ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see fig.). Show that:
 - $\triangle APB = \triangle CQD$
 - $AP = CQ$
- The diagonals of a quadrilateral ABCD are perpendicular to each other. Show that the quadrilateral formed by joining the mid-points of its sides is a rectangle.

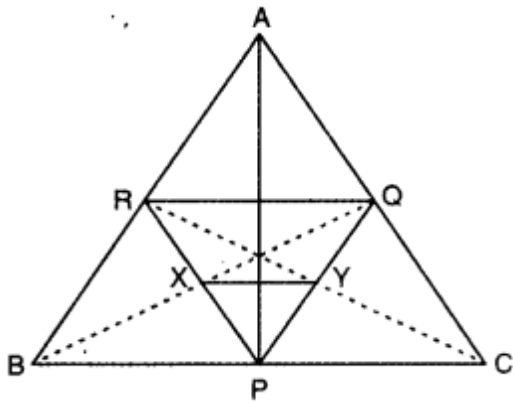


- In the fig., D, E and F are, respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC. Prove that DEF is also an equilateral triangle.

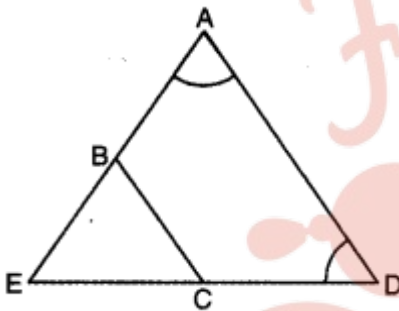


Long Questions:

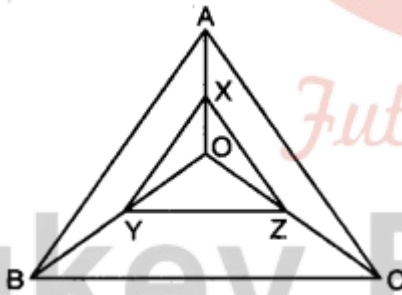
- In the figure, P, Q and R are the mid-points of the sides BC, AC and AB of $\triangle ABC$. If BQ and PR intersect at X and CR and PQ intersect at Y, then show that $XY = \frac{1}{4} BC$



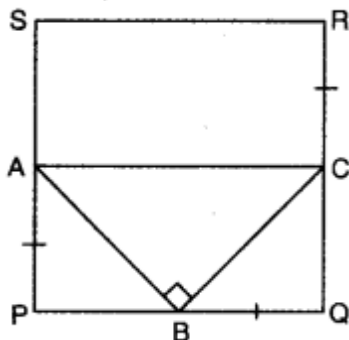
2. In the given figure, $AE = DE$ and $BC \parallel AD$. Prove that the points A, B, C and D are concyclic. Also, prove that the diagonals of the quadrilateral ABCD are equal.



3. In $\triangle ABC$, $AB = 8\text{cm}$, $BC = 9\text{cm}$ and $AC = 10\text{cm}$. X, Y and Z are mid-points of AO, BO and CO respectively as shown in the figure. Find the lengths of the sides of $\triangle XYZ$.



4. PQRS is a square and $\angle ABC = 90^\circ$ as shown in the figure. If $AP = BQ = CR$, then prove that $\angle BAC = 45^\circ$



5. ABCD is a parallelogram. If the bisectors DP and CP of angles D and C meet at P on side AB, then show that P is the mid-point of side AB.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- Assertion and reason both are correct statements and reason is correct explanation for assertion.
- Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- Assertion is correct statement but reason is wrong statement.
- Assertion is wrong statement but reason is correct statement.

Assertion: ABCD is a square. AC and BD intersect at O. The measure of $\angle AOB = 90^\circ$.

Reason: Diagonals of a square bisect each other at right angles.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- Assertion and reason both are correct statements and reason is correct explanation for assertion.
- Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- Assertion is correct statement but reason is wrong statement.
- Assertion is wrong statement but reason is correct statement.

Assertion: The consecutive sides of a quadrilateral have one common point.

Reason: The opposite sides of a quadrilateral have two common point.

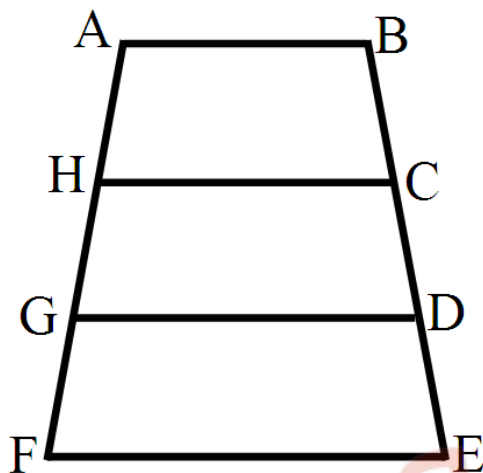
Case Study Questions-

1. Read the Source/ Text given below and answer these questions:



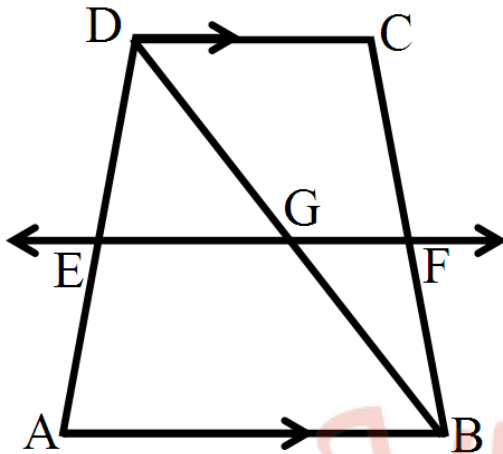
Sohan wants to show gratitude towards his teacher by giving her a card made by him. He has three pieces of trapezium pasted one above the other as shown in fig. These pieces are arranged in a way that $AB \parallel HC \parallel GD \parallel FE$. Also $BC =$

$CD = DE$ and $AH = HG = GF = 6\text{cm}$. He wants to decorate the card by putting up a colored tape on the nonparallel sides of the trapezium.



- i. Find the total length of colored tape required if $DE = 4\text{cm}$.
 - a. 20cm
 - b. 30cm
 - c. 40cm
 - d. 50cm
- ii. $ABHC$ is a trapezium in which $AB \parallel HC$ and $\angle A = \angle B = 45^\circ$. Find angles C and H of the trapezium.
 - a. 135, 130
 - b. 130, 135
 - c. 135, 135
 - d. 130, 130
- iii. What is the difference between trapezium and parallelogram?
 - a. Trapezium has 2 sides, and parallelogram has 4 sides.
 - b. Trapezium has 4 sides, and parallelogram has 2 sides.
 - c. Trapezium has 1 pair of parallel sides, and parallelogram has 2 pairs of parallel sides.
 - d. Trapezium has 2 pairs of parallel sides, and parallelogram has 1 pair of parallel sides.
- iv. Diagonals in isosceles trapezoid are _____.
 - a. parallel.
 - b. opposite.
 - c. vertical.
 - d. equal.

- v. ABCD is a trapezium where $AB \parallel DC$, BD is the diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F. Which of these is true?

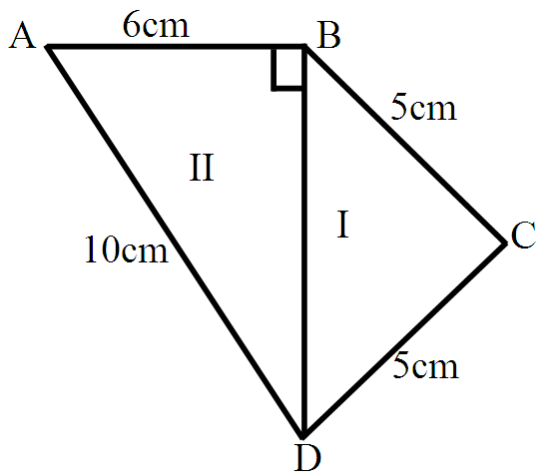


- $BF = FC$
- $EA = FB$
- $CF = DE$
- None of these

2. Read the Source/ Text given below and answer any four questions:



Chocolate is in the form of a quadrilateral with sides 6cm and 10cm, 5cm and 5cm (as shown in the figure) is cut into two parts on one of its diagonal by a lady. Part-I is given to her maid and part II is equally divided among a driver and gardener.



- i. Length of BD:
 - a. 9cm
 - b. 8cm
 - c. 7cm
 - d. 6cm
- ii. Area of $\triangle ABC$:
 - a. 24cm^2
 - b. 12cm^2
 - c. 42cm^2
 - d. 21cm^2
- iii. The sum of all the angles of a quadrilateral is equal to:
 - a. 180°
 - b. 270°
 - c. 360°
 - d. 90°
- iv. A diagonal of a parallelogram divides it into two congruent:
 - a. Square.
 - b. Parallelogram.
 - c. Triangles.
 - d. Rectangle.
- v. Each angle of the rectangle is:
 - a. More than 90°
 - b. Less than 90°
 - c. Equal to 90°

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d. Equal to 45°

Answer Key:

MCQ:

1. (b) 50°
2. (c) 68°
3. (c) Trapezium
4. (c) Rectangle
5. (b) Kite
6. (a) $115^\circ, 79^\circ, 118^\circ, 48^\circ$
7. (d) 10cm
8. (a) 175°
9. (d) 38°
- 10.(a) $\angle A = \angle B$

Very Short Answer:

1. Let the two adjacent angles be x and $2x$.

In a parallelogram, sum of the adjacent angles are 180°

$$\therefore x + 2x = 180^\circ$$

$$\Rightarrow 3x = 180^\circ$$

$$\Rightarrow x = 60^\circ$$

Thus, the two adjacent angles are 120° and 60° . Hence, the angles of the parallelogram are $120^\circ, 60^\circ, 120^\circ$ and 60° .

2. Rhombus.

3. Let each equal angle of given quadrilateral be x .

We know that sum of all interior angles of a quadrilateral is 360°

$$\therefore x + x + x + 144^\circ = 360^\circ$$

$$3x = 360^\circ - 144^\circ$$

$$3x = 216^\circ$$

$$x = 72^\circ$$

Hence, each equal angle of the quadrilateral is of 72o measures.

4. $\angle A - \angle C = 0^\circ$ (opposite angles of parallelogram are equal]

- 5.

$$12 \text{ cm}$$

Here, $PQ = SR = 12 \text{ cm}$

Let $PS = x$ and $PS = QR$

$$\therefore x + 12 + x + 12 = \text{Perimeter}$$

$$2x + 24 = 40$$

$$2x = 16$$

$$x = 8$$

Hence, length of each side of the parallelogram is 12cm, 8 cm, 12cm and 8cm.

6. We know that consecutive interior angles of a parallelogram are supplementary.

$$\therefore (x + 60^\circ + (2x + 30)^\circ = 180^\circ$$

$$\Rightarrow 3x^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow 3x^\circ = 90^\circ$$

$$\Rightarrow x^\circ = 30^\circ$$

Thus, two consecutive angles are $(30 + 60)^\circ, 12 \times 30 + 30)^\circ$. i.e., 90° and 90° .

Hence, the special name of the given parallelogram is rectangle.

7. Since ONKA is a square

$$\therefore \angle AON = 90^\circ$$

We know that diagonal of a square bisects its \angle s

$$\Rightarrow \angle AOK = \angle KON = 45^\circ$$

$$\text{Hence, } \angle KOA = 45^\circ$$

$$\text{Now, } \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + 70^\circ + 70^\circ = 180^\circ$$

$$[\because \angle B = 70^\circ]$$

$$\Rightarrow \angle A = 180^\circ - 70^\circ - 70^\circ = 40^\circ$$

8. Let $\angle Q = 2x, \angle R = 3x$ and $\angle S = 7x$

$$\text{Now, } \angle P + \angle Q + \angle R + \angle S = 360^\circ$$

$$\Rightarrow 60^\circ + 2x + 3x + 7x = 360^\circ$$

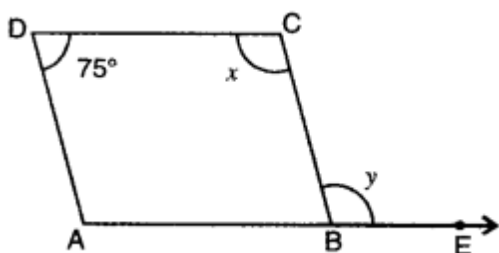
$$\Rightarrow 12x = 300^\circ$$

$$x = \frac{300^\circ}{12} = 25^\circ$$

$$\angle S = 7x = 7 \times 25^\circ = 175^\circ$$

Short Answer:

Ans: 1.



Here, $\angle C$ and $\angle D$ are adjacent angles of the parallelogram.

$$\therefore \angle C + \angle D = 180^\circ$$

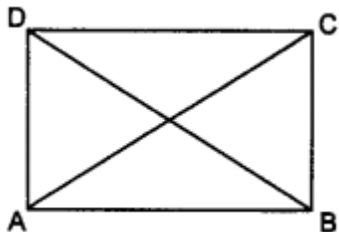
$$\Rightarrow x + 75^\circ = 180^\circ$$

$$\Rightarrow x = 105^\circ$$

Also, $y = x = 105^\circ$ [alt. int. angles]

$$\text{Thus, } x + y = 105^\circ + 105^\circ = 210^\circ$$

Ans: 2.



Given: A parallelogram ABCD, in which $AC = BD$.

To Prove: $\triangle BCD$ is a rectangle.

Proof: In $\triangle ABC$ and $\triangle BAD$

$$AB = AB \text{ (common)}$$

$$AC = BD \text{ (given)}$$

$$BC = AD \text{ (opp. sides of a || gm)}$$

$$\Rightarrow \triangle ABC \cong \triangle BAD$$

[by SSS congruence axiom]

$$\Rightarrow \angle ABC = \angle BAD \text{ (c.p.c.t.)}$$

$$\text{Also, } \angle ABC + \angle BAD = 180^\circ \text{ (co-interior angles)}$$

$$\angle ABC + \angle ABC = 180^\circ \text{ [} \because \angle ABC = \angle BAD \text{]}$$

$$2\angle ABC = 180^\circ$$

$$\angle ABC = \frac{1}{2} \times 180^\circ = 90^\circ$$

Hence, parallelogram ABCD is a rectangle.

Ans: 3. Since diagonals of a rhombus bisect each other at right angle.

In $\therefore \triangle AOB$, we have

$$\angle OAB + \angle x + 90^\circ = 180^\circ$$

$$\angle x = 180^\circ - 90^\circ - 35^\circ$$

$$= 55^\circ$$

Also,

$$\angle DAO = \angle BAO = 35^\circ$$

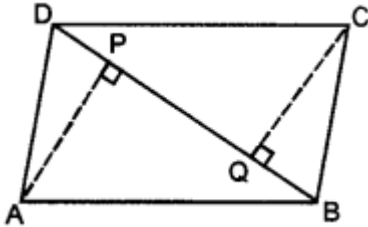
$$\angle y + \angle DAO + \angle BAO + \angle x = 180^\circ$$

$$\Rightarrow \angle y + 35^\circ + 35^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle y = 180^\circ - 125^\circ = 55^\circ$$

Hence, the values of x and y are $x = 55^\circ$, $y = 55$

Ans: 4.



Given: In $\parallel\text{gm}$ ABCD, AP and CQ are perpendiculars from the vertices A and C on the diagonal BD.

To Prove: (i) $\triangle APB \cong \triangle CQD$

(ii) $AP = CQ$

Proof: (i) In $\triangle APB$ and $\triangle CQD$

$AB = DC$ (opp. sides of a $\parallel\text{gm}$ ABCD)

$\angle APB = \angle CQD$ (each $= 90^\circ$)

$\angle ABP = \angle CDQ$ (alt. int. \angle s)

$\Rightarrow \triangle APB \cong \triangle CQD$ [by AAS congruence axiom]

(ii) $\Rightarrow AP = CQ$ [c.p.c.t.]

Ans: 5. Given: A quadrilateral ABCD whose diagonals AC and BD are perpendicular to each other at O. P, Q, R and S are mid-points of side AB, BC, CD and DA respectively are joined are formed quadrilateral PQRS.

To Prove: PQRS is a rectangle.

Proof: In $\triangle ABC$, P and Q are mid-points of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$... (i) (mid-point theorem)

Further, in $\triangle ACD$, R and S are mid-points of CD and DA respectively.

$SR \parallel AC$ and $SR = \frac{1}{2} AC$... (ii) (mid-point theorem)

From (i) and (ii), we have $PQ \parallel SR$ and $PQ = SR$

Thus, one pair of opposite sides of quadrilateral PQRS are parallel and equal.

\therefore PQRS is a parallelogram.

Since $PQ \parallel AC$ and $SR \parallel AC$

In $\triangle ABD$, P and S are mid-points of AB and AD respectively.

$PS \parallel BD$ (mid-point theorem)

$$\Rightarrow PN \parallel MO$$

\therefore Opposite sides of quadrilateral PMON are parallel.

\therefore PMON is a parallelogram.

$\angle MPN = \angle MON$ (opposite angles of \parallel gm are equal]

But $\angle MON = 90^\circ$ [given]

$$\therefore \angle MPN = 90^\circ \Rightarrow \angle QPS = 90^\circ$$

Thus, PQRS is a parallelogram whose one angle is 90°

\therefore PQRS is a rectangle.

Ans: 6. Since line segment joining the mid-points of two sides of a triangle is half of the third side.

Therefore, D and E are mid-points of BC and AC respectively.

$$\Rightarrow DE = \frac{1}{2} AB \dots (i)$$

E and F are the mid-points of AC and AB respectively.

$$\therefore EF = \frac{1}{2} BC \dots (ii)$$

F and D are the mid-points of AB and BC respectively.

$$\therefore FD = \frac{1}{2} AC \dots (iii)$$

Now, $\triangle ABC$ is an equilateral triangle.

$$\Rightarrow AB = BC = CA$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$$

$$\Rightarrow DE = EF = FD \text{ (using (i), (ii) and (iii))}$$

Hence, DEF is an equilateral triangle

Long Answer:

Ans: 1. Here, in $\triangle ABC$, R and Q are the mid-points of AB and AC respectively.

\therefore By using mid-point theorem, we have

$$RQ \parallel BC \text{ and } RQ = \frac{1}{2} BC$$

$\therefore RQ = BP = PC$ [\because P is the mid-point of BC]

$\therefore RQ \parallel BP$ and $RQ \parallel PC$

In quadrilateral BPQR

$RQ \parallel BP$, $RQ = BP$ (proved above]

\therefore BPQR is a parallelogram. [\because one pair of opp. sides is parallel as well as equal]

\therefore X is the mid-point of PR. [\because diagonals of a \parallel gm bisect each other]

Now, in quadrilateral PCQR

$RQ \parallel PC$ and $RQ = PC$ [proved above]

\therefore PCQR is a parallelogram [\because one pair of opp. sides is parallel as well as equal]

\therefore Y is the mid-point of PQ [\because diagonals of a \parallel gm bisect each other]

In ΔPQR

\therefore X and Y are mid-points of PR and PQ respectively.

$\therefore XY \parallel RQ$ and $XY = \frac{1}{2}RQ$ [by using mid-point theorem]

$$XY = \frac{1}{2} \left(\frac{1}{2}BC \right) \quad [\because RQ = \frac{1}{2}BC]$$

$$\Rightarrow XY = \frac{1}{4}BC$$

Ans: 2. Since $AE = DE$

$\angle D = \angle A$ (i) [\because \angle s opp. to equal sides of a Δ]

Again, $BC \parallel AD$

$\angle EBC = \angle A$ (ii) (corresponding \angle s)

From (i) and (ii), we have

$\angle D = \angle EBC$ (iii)

But $\angle EBC + \angle ABC = 180^\circ$ (a linear pair)

$\angle D + \angle ABC = 180^\circ$ (using (iii))

Now, a pair of opposite angles of quadrilateral ABCD is supplementary

Thus, ABCD is a cyclic quadrilateral i.e., A, B, C and D are concyclic. In ΔABD and ΔDCA

$\angle ABD = \angle ACD$ [\angle s in the same segment for cyclic quad. ABCD]

$\angle BAD = \angle CDA$ [using (i)]

$AD = AD$ (common)

So, by using AAS congruence axiom, we have

$\Delta ABD \cong \Delta DCA$

Hence, $BD = CA$ [c.p.c.t.]

Ans: 3. Here, in ΔABC , $AB = 8\text{cm}$, $BC = 9\text{cm}$, $AC = 10\text{cm}$.

In ΔAOB , X and Y are the mid-points of AO and BO.

\therefore By using mid-point theorem, we have

$$XY = \frac{1}{2} AB = \frac{1}{2} \times 8\text{cm} = 4\text{cm}$$

Similarly, in ΔBOC , Y and Z are the mid-points of BO and CO.

\therefore By using mid-point theorem, we have

$$YZ = \frac{1}{2} BC = \frac{1}{2} \times 9\text{cm} = 4.5\text{cm}$$

And, in ΔCOA , Z and X are the mid-points of CO and AO.

$$\therefore ZX = \frac{1}{2} AC = \frac{1}{2} \times 10\text{cm} = 5\text{cm}$$

Hence, the lengths of the sides of $\triangle XYZ$ are $XY = 4\text{cm}$, $YZ = 4.5\text{ cm}$ and $ZX = 5\text{cm}$.

Ans: 4. Since PQRS is a square.

$\therefore PQ = QR \dots$ (i) [\because sides of a square are equal]

Also, $BQ = CR \dots$ (ii) [given]

Subtracting (ii) from (i), we obtain

$$PQ - BQ = QR - CR$$

$$\Rightarrow PB = QC \dots$$
 (iii)

In $\triangle APB$ and $\triangle BQC$

$$AP = BQ$$

[given $\angle APB = \angle BQC = 90^\circ$](each angle of a square is 90°)

$$PB = QC \text{ (using (iii))}$$

So, by using SAS congruence axiom, we have

$$\triangle APB \cong \triangle BQC$$

$$\therefore AB = BC \text{ [c.p.c.t.]}$$

Now, in $\triangle ABC$

$$AB = BC \text{ [proved above]}$$

$$\therefore \angle ACB = \angle BAC = x^\circ \text{ (say) } [\angle\text{s opp. to equal sides}]$$

$$\text{Also, } \angle B + \angle ACB + \angle BAC = 180^\circ$$

$$\Rightarrow 90^\circ + x + x = 180^\circ$$

$$\Rightarrow 2x^\circ = 90^\circ$$

$$x^\circ = 45^\circ$$

$$\text{Hence, } \angle BAC = 45^\circ$$

Ans: 5.

U

D

Since DP and CP are angle bisectors of $\angle D$ and $\angle C$ respectively.

$$\therefore \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$$

Now, $AB \parallel DC$ and CP is a transversal

$$\therefore \angle 5 = \angle 1 \text{ [alt. int. } \angle\text{s]}$$

$$\text{But } \angle 1 = \angle 2 \text{ [given]}$$

$$\therefore \angle 5 = \angle 2$$

Now, in $\triangle BCP$, $\angle 5 = \angle 2$

$$\Rightarrow BC = BP \dots$$
 (I) [sides opp. to equal \angle s of a \triangle]

Again, $AB \parallel DC$ and DP is a transversal.

$$\therefore \angle 6 = \angle 3 \text{ (alt. int. } \angle\text{s)}$$

$$\text{But } \angle 4 = \angle 3 \text{ [given]}$$

$$\therefore \angle 6 = \angle 4$$

Now, in $\triangle ADP$, $\angle 6 = \angle 4$

$\Rightarrow DA = AP \dots$ (ii) (sides opp. to equal \angle s of a Δ)

Also, $BC = DA \dots$ (iii) (opp. sides of parallelogram)

From (i), (ii) and (iii), we have

$$BP = AP$$

Hence, P is the mid-point of side AB.

Assertion and Reason Answers-

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
2. c) Assertion is correct statement but reason is wrong statement.

Case Study Answers-

1.

(i)	(b)	30cm
(ii)	(c)	135, 135
(iii)	(c)	Trapezium has 1 pair of parallel sides, and parallelogram has 2 pairs of parallel sides.
(iv)	(d)	equal.
(v)	(a)	$BF = FC$

2.

(i)	(b)	8cm
(ii)	(a)	24cm^2
(iii)	(c)	360°
(iv)	(c)	Triangles.
(v)	(c)	Equal to 90°

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