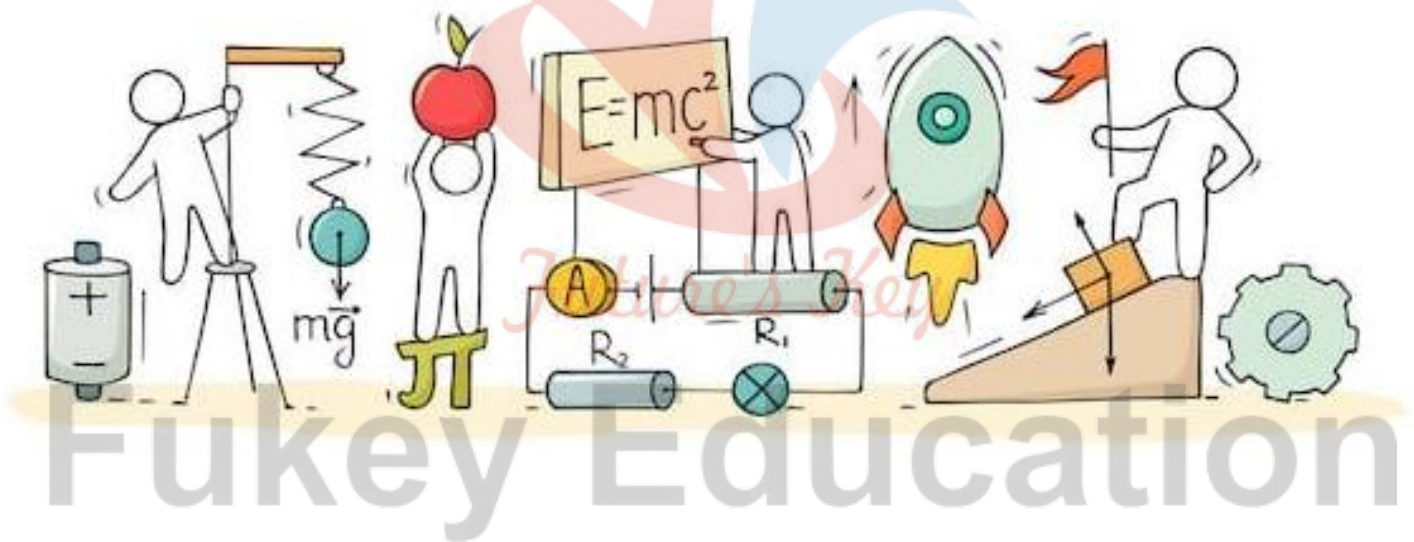


# PHYSICS

## Chapter 6: System of Particles and Rotational Motion



## System of Particles and Rotational Motion

### Introduction

Rigid body is a body with a perfectly definite and unchanging shape. The distances between all pairs of particles of such a body do not change.



Bridge can be taken as rigid body as the distance between any two particles does not change or it is so small that it can be neglected



The lemon is not a rigid body since we can see significant change in its shape after application of force

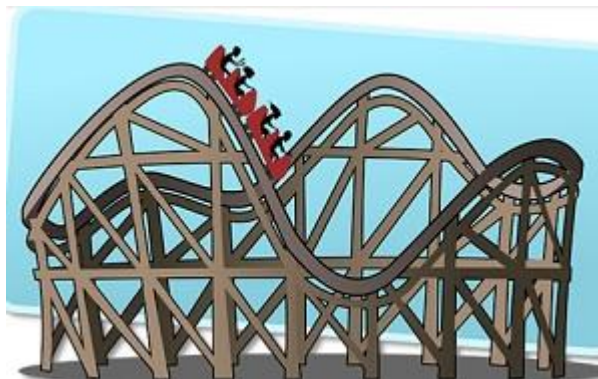
In pure translational motion at any instant of time all particles of the body have the same velocity.



In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.



The motion of a rigid body which is not pivoted or fixed in some way is either a pure translation or a combination of translation and rotation. The motion of a rigid body which is pivoted or fixed in some way is rotation.





## Rotational Motion

Let's us understand this by an example. Now let us imagine a circular block going down the edge of the right-angled triangle. Examining the location and orientation of different points on the cylindrical block will tell us something new. The points on the cylindrical body experience something much different than the rectangular block. As shown by the arrows in the diagram representing the velocity, each point experiences a different magnitude of velocity in a different direction. Here the points are arranged with respect to an axis of rotation.

Rotation is what you achieve when you constrain a body and fix it along with a straight line. This means that the body can only turn around the line, which is defined as rotational motion. A ceiling fan, a potter's wheel, a vehicle's wheel are all examples of rotational motion.

## Translational Motion

Let us understand translational motion with the help of examples. Let's imagine a rectangular block placed on the slanting edge of a right-angled triangle. If the block is assumed to slide down this edge without any side movement, every point in the rectangular block experiences the same displacement and more importantly, the distance between the points is also maintained.

In a pure translational motion, every point in the body experiences the same velocity be it at any instant of time. Both the points, P1 and P2 undergo the exact same motions. A car moving in a straight line, the path of a bullet out of a gun etc. are examples of translational motion.

## Centre of mass

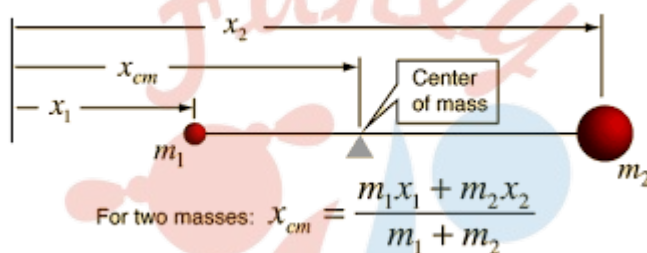
Centre of mass of a body or system of a particle is defined as, a point at which the whole of the mass of the body or all the masses of a system of particle appeared to be concentrated. In physics, we can say that the centre of mass is a point at the centre of the distribution of mass in space (also known as balance point) wherein the weighted relative position of the distributed mass has a sum of zero. In simple words, the centre of mass is a position that is relative to an object. We can say that it is the average position of all the parts of the system or it is the mean



location of a distribution of mass in space. It is a point where force is usually applied that results in linear acceleration without any angular acceleration.

When we are studying the dynamics of the motion of the system of a particle as a whole, then we need not bother about the dynamics of individual particles of the system. But only focus on the dynamic of a unique point corresponding to that system.

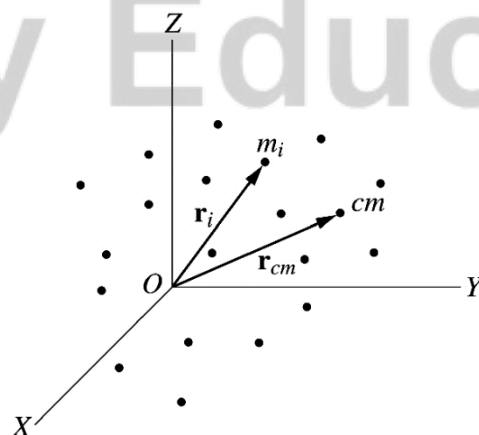
Motion of this unique point is identical to the motion of a single particle whose mass is equal to the sum of all individual particles of the system and the resultant of all the forces exerted on all the particles of the system by surrounding bodies (or) action of a field of force is exerted directly to that particle. This point is called the centre of mass of the system of particles. The concept of centre of mass (COM) is useful in analyzing the complicated motion of the system of objects, particularly when two and more objects collide, or an object explodes into fragments.



### Centre of Gravity

The Centre of gravity can be taken as the point through which the force of gravity acts on an object or system. It is basically the point around which the resultant torque due to gravity forces disappears. In cases where the gravitational field is assumed to be uniform, the centre of gravity and centre of mass will be the same. Sometimes these two terms – the centre of gravity and centre of mass are used interchangeably as they are often said to be at the same position or location.

### System of Particles



The term system of particles means a well-defined collection of a large number of particles that may or may not interact with each other or are connected to each other. They may be actual particles of rigid bodies in translational motion. The particle which interacts with each other apply force on each other.

The force of interaction  $\vec{F}_{ij}$  and  $\vec{F}_{ji}$  between a pair of  $i^{\text{th}}$  and  $j^{\text{th}}$  particle.

These forces of mutual interaction between the particle of the system are called the internal force of the system.

These internal forces always exist in pairs of equal magnitude and opposite directions. Other than internal forces, external forces may also act on all or some of the particles. Here the term external force means a force that is acting on any one particle, which is included in the system by some other body outside the system.

### Rigid body

In practice, we deal with extended bodies, which may be deformable or non-deformable (or) rigid. An extended body is also a system of an infinitely large number of particles having an infinitely small separation between them. When a body deforms, the separation between the distance between its particles and their relative locations changes. A rigid body is an extended object in which the separations and relative location of all of its constituent particles remain the same under all circumstances.

It is the average position of all the parts of the system, weighted according to their masses. For a simple rigid object which has a uniform density, the centre of mass is located at the centroid.

### Linear Momentum of a System of Particles

Linear momentum is a product of the mass ( $m$ ) of an object and the velocity ( $v$ ) of the object. If an object has higher momentum, then it harder to stop it. The formula for linear momentum is  $p = mv$ . The total amount of momentum never changes, and this property is called conservation of momentum. Let us study more about Linear momentum and conservation of momentum.



We know that the linear momentum of the particle is

$$p = mv$$

Newton's second law for a single particle is given by,

$$F = \frac{dP}{dt}$$

where  $F$  is the force of the particle. For 'n' no. of particles total linear momentum is,

$$P = p_1 + p_2 + \dots + p_n$$

each of momentum is written as  $m_1 v_1 + m_2 v_2 + \dots + m_n v_n$ . We know that velocity of the centre of mass is  $V = \Sigma \frac{m_i v_i}{M}$ ,

$$mv = \Sigma m_i v_i$$

So comparing these equations we get,

$$P = M V$$

Therefore we can say that the total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its center of mass. Differentiating the above equation we get,

$$\frac{dP}{dt} = M \frac{dV}{dt} = MA$$

$\frac{dv}{dt}$  is acceleration of centre of mass,  $MA$  is the force external. So,

$$\frac{dP}{dt} = F_{\text{ext}}$$

This above equation is nothing but Newton's second law to a system of particles. If the total external force acting on the system is zero,

$$F_{\text{ext}} = 0 \text{ then, } \frac{dP}{dt} = 0$$

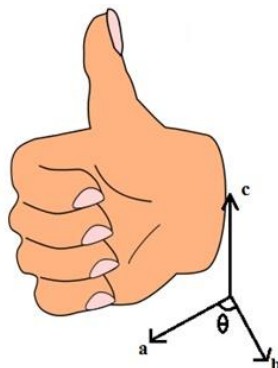
This means that  $P = \text{constant}$ . So, whenever the total force acting on the system of a particle is equal to zero then the total linear momentum of the system is constant or conserved. This is nothing but the law of conservation of total linear momentum of a system of particles.

### Vector Product

Vector product (cross product) of two vectors  $a$  and  $b$  is  $a \times b = ab \sin\theta = c$ , where  $\theta$  is angle between  $a$  &  $b$

Vector product  $c$  is perpendicular to the plane containing  $a$  and  $b$ .

If you keep your palm in direction of vector  $a$  and curl your fingers to the direction  $a$  to  $b$ , your thumb will give you the direction of vector product  $c$



### Properties of vector product

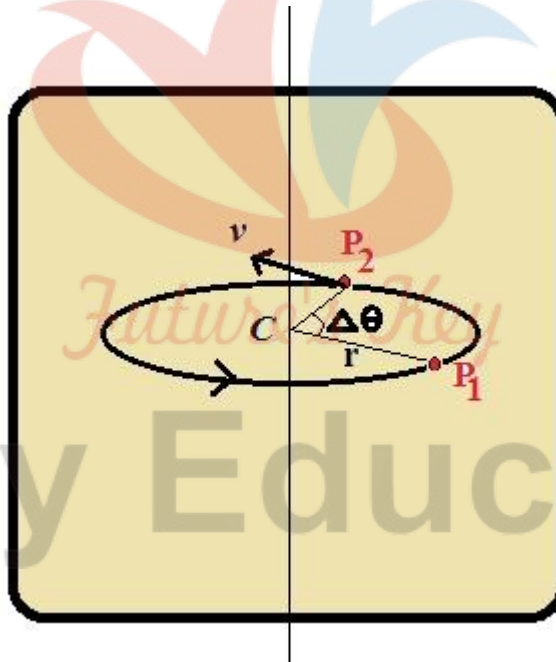
- $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$
- $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
- $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
- $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ ,  $\mathbf{0}$  is called null vector i.e having zero magnitude
- $\hat{i} \times \hat{j} = \hat{k}$
- $\hat{j} \times \hat{k} = \hat{i}$
- $\hat{k} \times \hat{i} = \hat{j}$

$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$  and  $\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$  then their vector product is given by

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

### Angular velocity & its relation with linear velocity

Every particle of a rotating body moves in a circle. Angular displacement of a given particle about its centre in unit time is defined as angular velocity.



$$\text{Average angular velocity} = \frac{\Delta\theta}{\Delta t}$$

$$\text{Instantaneous angular velocity, } \omega = \frac{d\theta}{dt}$$

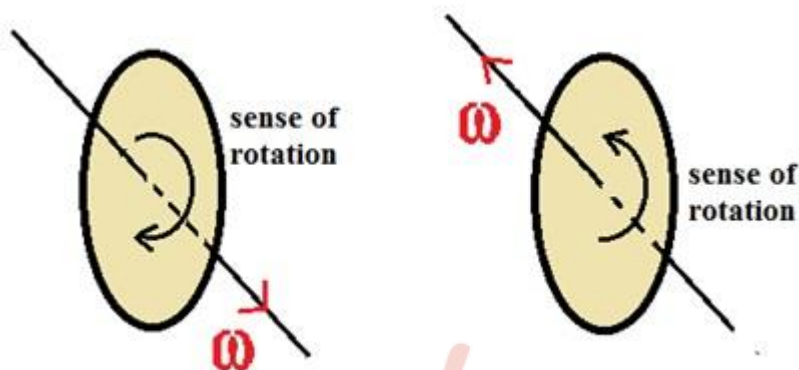
$v = \omega r$ , where  $v$  – linear velocity of particle moving in a circle of radius  $r$

All parts of a moving body have the same angular velocity in pure rotation motion.

Angular velocity,  $\omega$ , is a vector quantity



If you curl your fingers of right hand in the sense of rotation, thumb will give direction of angular velocity.



$$v = \omega \times r$$

Angular acceleration is given by rate of change of angular velocity with respect to time.

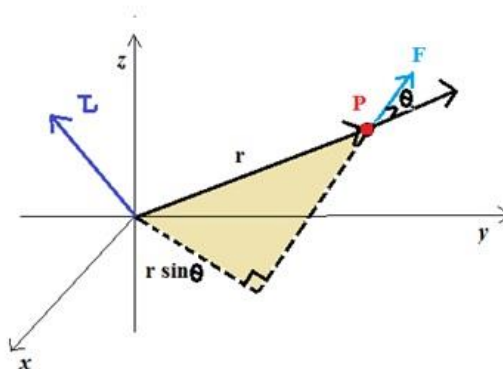
$$\alpha = \frac{d\omega}{dt}$$



## Torque & Angular Momentum

The rotational analogue of force is moment of force (Torque).

If a force acts on a single particle at a point P whose position with respect to the origin O is given by the position vector  $r$  the moment of the force acting on the particle with respect to the origin O is defined as the vector product  $\mathbf{t} = \mathbf{r} \times \mathbf{F} = rF \sin\theta$



Torque is vector quantity.

The moment of a force vanishes if either

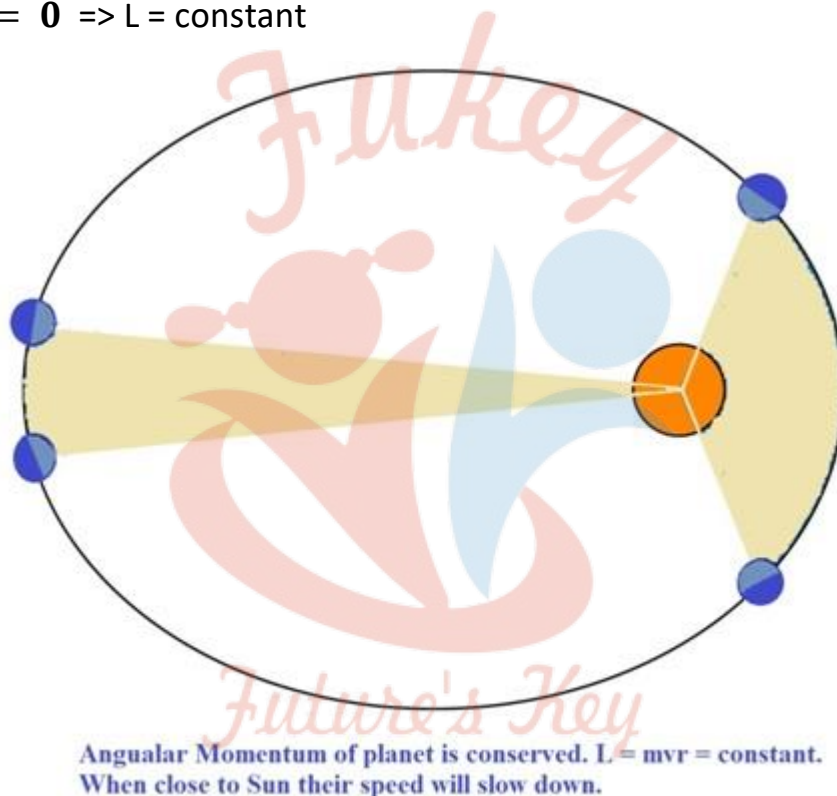
The magnitude of the force is zero, or

The line of action of the force ( $r \sin\theta$ ) passes through the axis.

### Conservation of Angular Momentum

if the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved

If  $\tau_{ext} = 0$ , then  $\frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$



### Equilibrium of Rigid Body

A force changes the translational state of the motion of the rigid body, i.e., it changes its total linear momentum.

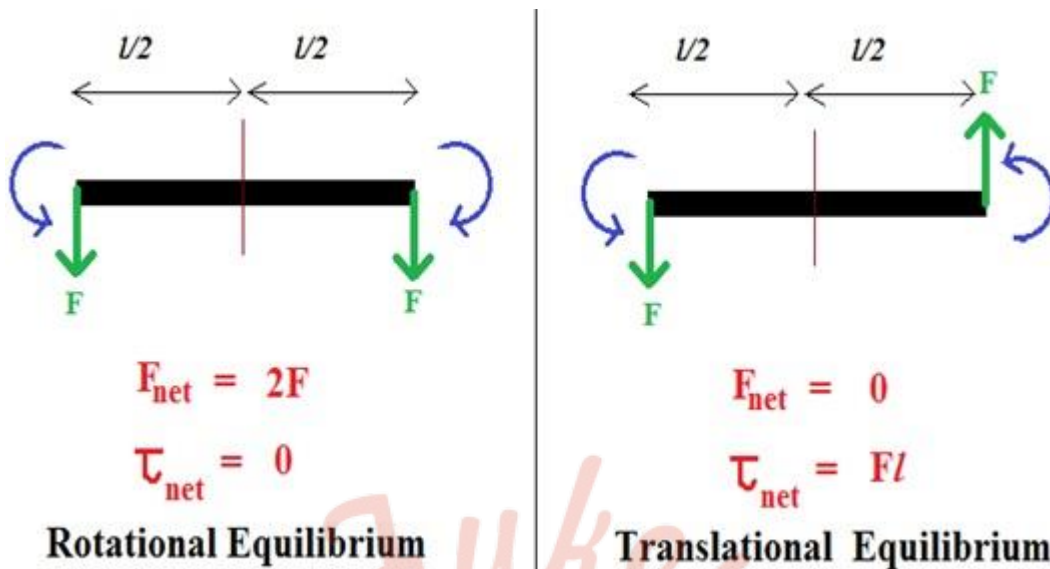
A torque changes the rotational state of motion of the rigid body, i.e., it changes the total angular momentum of the body

Note: Unless stated otherwise, we shall deal with only external forces and torques.

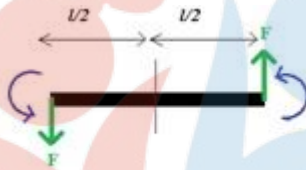
A rigid body is said to be in mechanical equilibrium, if both its linear momentum and angular momentum are not changing with time. This means

Total force should be zero  $\Rightarrow$  Translational Equilibrium

Total torque should be zero  $\Rightarrow$  Rotational Equilibrium



A pair of equal and opposite forces with different lines of action is known as a couple or torque. A couple produces rotation without translation.

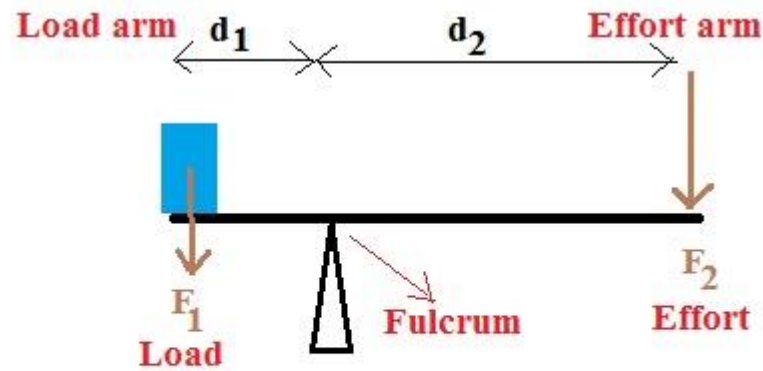


When you open the lid of a jar, you apply couple on it



An ideal lever is essentially a light rod pivoted at a point along its length. This point is called the fulcrum

The lever is a system in mechanical equilibrium.



$$\text{Mechanical Advantage} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

Mechanical advantage greater than one means that a small effort can be used to lift a large load.

### Centre of Gravity

The centre of gravity of a body is that point where the total gravitational torque on the body is zero.



The centre of gravity of the body coincides with the centre of mass in uniform gravity or gravity-free space.

If  $g$  varies from part to part of the body, then the centre of gravity and centre of mass will not coincide.

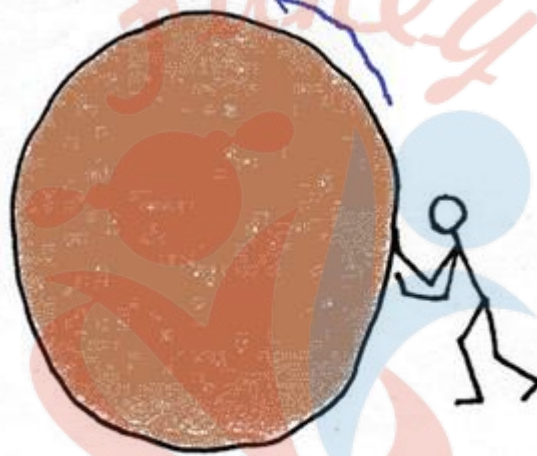
### Moment of Inertia

- Moment of inertia ( $I$ ) is analogue of mass in rotational motion.

$$I = \sum_{i=1}^n m_i r_i^2$$



- Moment of inertia about a given axis of rotation resists a change in its rotational motion; it can be regarded as a measure of rotational inertia of the body.
- It is a measure of the way in which different parts of the body are distributed at different distances from the axis.
- the moment of inertia of a rigid body depends on
  - The mass of the body,
  - Its shape and size
  - Distribution of mass about the axis of rotation
  - The position and orientation of the axis of rotation.



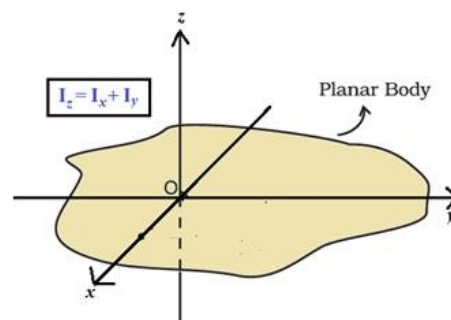
Why rolling such a huge stone is difficult than rolling a small coin?

Due to stone's large moment of inertia.

- The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.
- $I = M k^2$ , where  $k$  is radius of gyration.

### Theorem of perpendicular axis

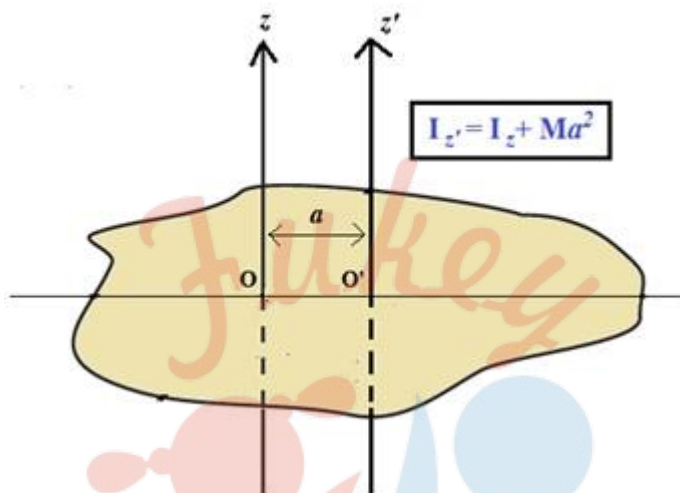
Perpendicular Axis Theorem: The moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.



Applicable only to planar bodies.

### Theorem of parallel axis

**Parallel Axis Theorem:** The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

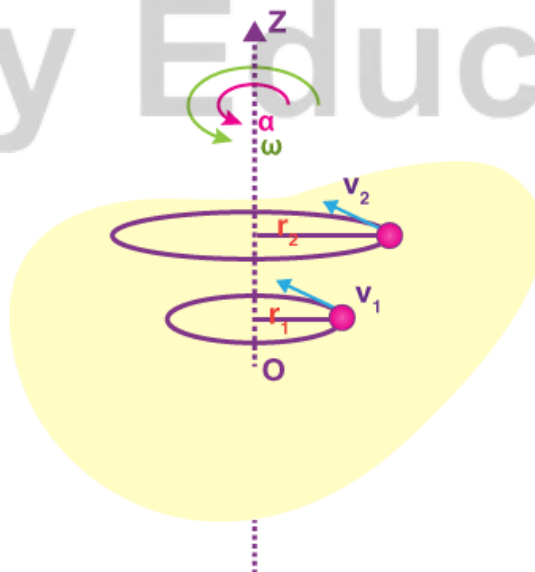


This theorem is applicable to a body of any shape.

### Kinematics of Rotational Motion around a Fixed Axis

As we know, the rotational motion and translational motion are analogous to each other in any respect. Also, the terms we use in rotational motion such as the angular velocity and angular acceleration are analogous to the terms velocity and acceleration in translational motion. In that respect, we see that the rotation of a body about a fixed axis is analogous to the linear motion of a body in translational motion. In this section, we will discuss the kinematics of a body undergoing rotational motion about a fixed axis.

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## Dynamics of Rotational Motion about a Fixed Axis

Only those components of torques, which are along the direction of the fixed axis, need to be considered because the component of the torque perpendicular to the axis of rotation will tend to turn the axis from its position.

This means

We need to consider only those forces that lie in planes perpendicular to the axis. Forces which are parallel to the axis will give torques perpendicular to the axis.

We need to consider only those components of the position vectors which are perpendicular to the axis. Components of position vectors along the axis will result in torques perpendicular to the axis

- Work done by torque is given by:  $dW = \tau d\theta$
- Power (instantaneous) is given by:  $P = \frac{dW}{dt} = \tau \omega$
- Kinetic Energy is given by:  $K = \frac{1}{2} I \omega^2$
- The rate of increase of kinetic energy is  $\frac{d}{dt} \left( \frac{I\omega^2}{2} \right) = I\omega \frac{d\omega}{dt}$   
(This is considering the moment of inertia does not change with time.)
- Since  $\alpha = \frac{d\omega}{dt}$ ,  $\frac{d}{dt} \left( \frac{I\omega^2}{2} \right) = I\omega\alpha$
- We know that Work Done is equal to Change in Kinetic Energy,  $\tau = I\alpha$
- The angular acceleration is directly proportional to the applied torque and is inversely proportional to the moment of inertia of the body.
- $\tau = I\alpha$  can be called as Newton's second law for rotation about a fixed axis.

Linear Motion	Rotational Motion about a Fixed Axis
1 Displacement $x$	Angular displacement $\theta$
2 Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3 Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4 Mass $M$	Moment of Inertia $I$
5 Force $F = Ma$	Torque $\tau = I\alpha$
6 Work $dW = F ds$	Work $W = \tau d\theta$
7 Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
8 Power $P = Fv$	Power $P = \tau\omega$
9 Linear momentum $p = Mv$	Angular momentum $L = I\omega$



### Angular Momentum in Case of Rotation about a Fixed Axis

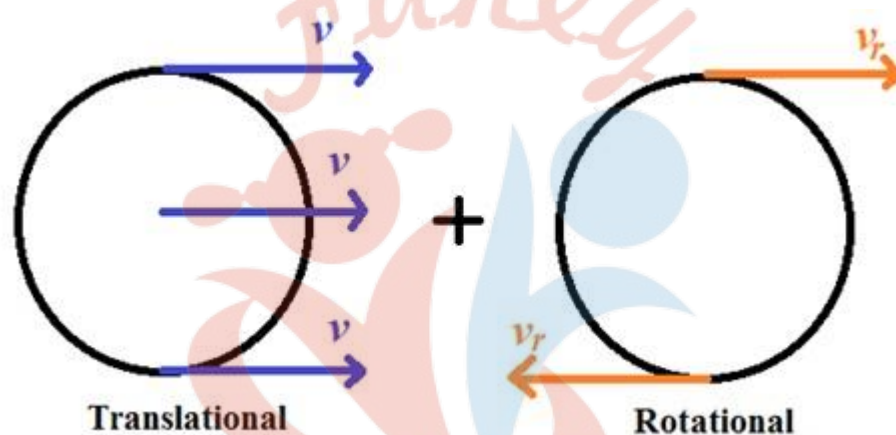
- We know that  $L = r \times p$   
 $L = r \times (mv)$   
 Now,  $v = \omega r \Rightarrow L = mr^2 \omega$
- $L = I\omega$

#### Conservation of angular momentum

- We know that,  $\frac{dL}{dt} = \frac{d}{dt}(I\omega) = \tau$
- If  $\tau_{\text{ext}} = 0$  then  $I\omega = \text{constant}$

### Rolling motion

Rolling motion is a combination of rotation and translation.

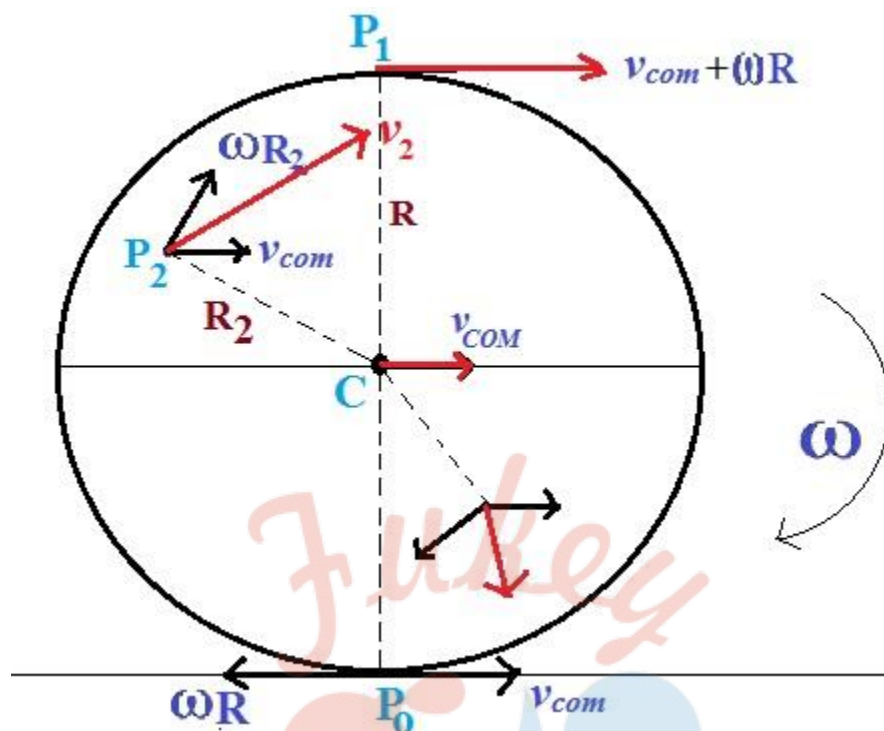


All the particles on a rolling body have two kinds of velocity

- Translational, which is velocity of COM.
- Linear velocity on account of rotational motion.







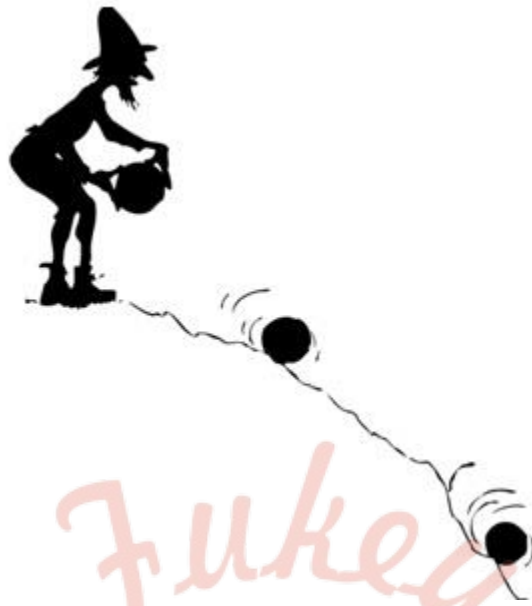
- Here in the figure we can see that every point have two velocities, one in the direction of velocity of COM and other perpendicular to the line joining centre and the point.
- Point  $P_0$  have opposite velocities, and if condition of no-slipping is there then it must have zero velocity, so  $v_{COM} = \omega R$
- At point  $P_1$  both the velocities add up.
- At any other point, add both the velocities vectorially to get the resultant, which are shown for some of the cases in red color in figure.
- The line passing through  $P_0$  and parallel to  $\omega$  is called the instantaneous axis of rotation.
- The point  $P_0$  is instantaneously at rest.
- Kinetic Energy of Rolling Motion
- $KE_{rolling} = KE_{translation} + KE_{rotation}$

$$KE = \frac{1}{2} I \omega^2 + \frac{1}{2} m v_{COM}^2$$

Substituting  $I = mk^2$  (where  $k$  is radius of gyration) and  $v_{COM} = R \omega$

We get

$$KE = \frac{1}{2} m v_{COM}^2 \left( 1 + \frac{k^2}{R^2} \right)$$



Top Formulae



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Position vector of COM of a system	$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots}$
Coordinates of COM	$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ $y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ $z = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \dots}{m_1 + m_2 + m_3 + \dots}$
Velocity of COM of a system of two particles	$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$
Equations of rotational motion	i) $\omega = \omega_1 + \alpha t$ ii) $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$ iii) $\omega_2^2 - \omega_1^2 = 2\alpha\theta$
Centripetal acceleration	$= \frac{v^2}{r} = r\omega^2$
Linear acceleration	$a = r\alpha$
Angular momentum	$\vec{L} = \vec{r} \times \vec{p}$
Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Kinetic energy of rotation	$= \frac{1}{2} I\omega^2$
Kinetic energy of translation	$= \frac{1}{2} mv^2$
Total kinetic energy	$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$
Angular momentum	$L = I\omega$
Torque	$\tau = I\alpha$
Relation between torque and angular momentum	$\vec{\tau} = \frac{d\vec{L}}{dt}$
Moment of inertia in terms of radius of gyration	$I = \sum_{i=1}^{i=n} m_i r_i^2 = MK^2$

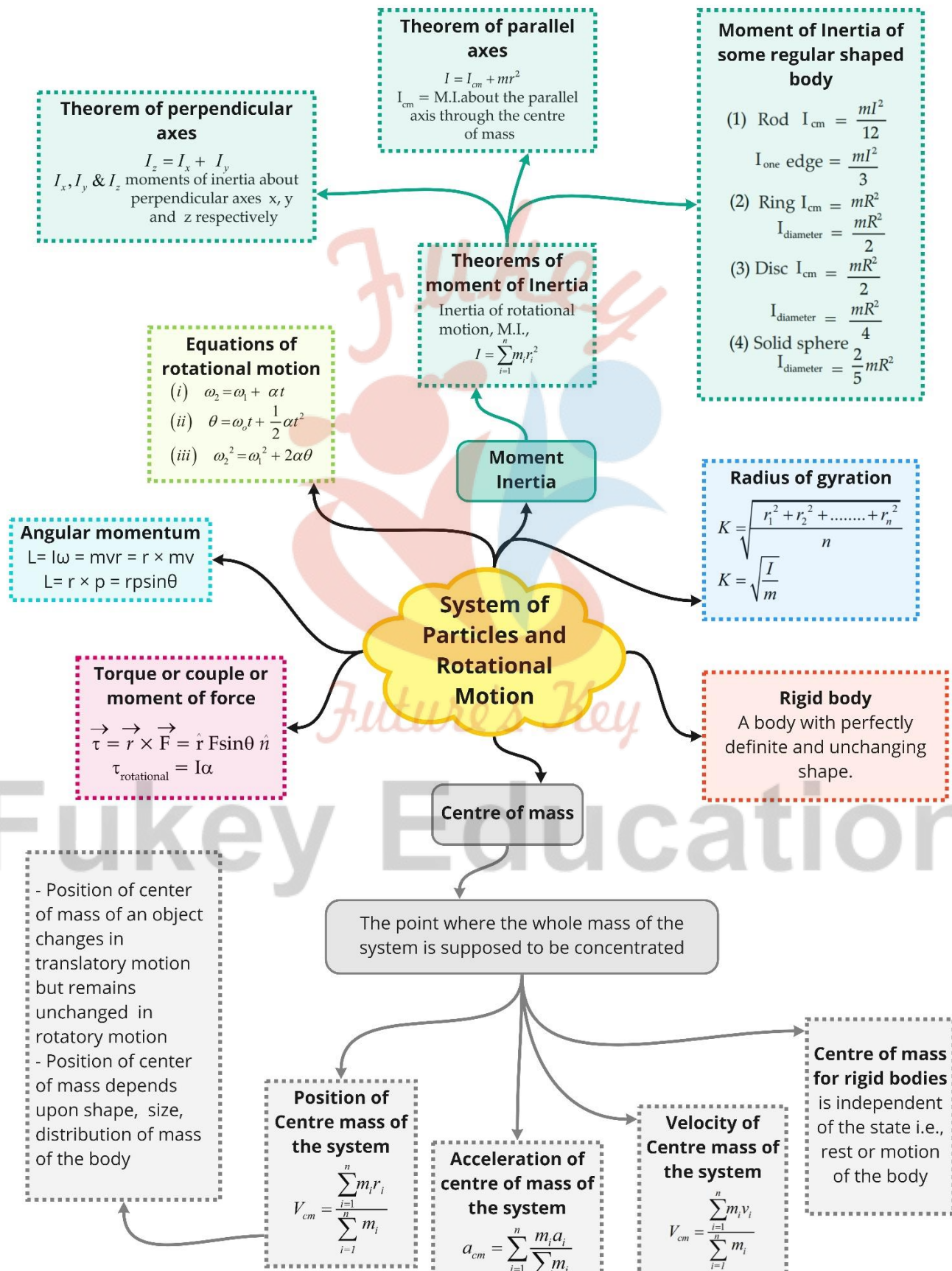
<b>Moment of inertia of a uniform circular ring about an axis passing through the centre and perpendicular to the plane of the ring</b>	$I = MR^2$
<b>For a uniform circular disc</b>	$I = \frac{1}{2}MR^2$
<b>For a thin uniform rod</b>	$I = \frac{1}{12}M\ell^2$
<b>For a hollow cylinder about its axis</b>	$I = MR^2$
<b>For a solid cylinder about its axis</b>	$I = \frac{1}{2}MR^2$
<b>For a hollow sphere about its diameter</b>	$I = \frac{2}{3}MR^2$
<b>For a solid sphere about its diameter</b>	$I = \frac{2}{5}MR^2$
<b>Coefficient of friction for rolling of solid cylinder without slipping down the rough inclined plane</b>	$\mu = \frac{1}{3}\tan\theta$

*Future's Key*

# Fukey Education



Class : 11th Physics  
Chapter- 7 : System of Particles and Rotational Motion



## Important Questions

### Multiple Choice questions-

1. A particle performing uniform circular motion has angular momentum  $L$ . If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is
  - (a)  $L/2$
  - (b)  $L/4$
  - (c)  $2L$
  - (d)  $4L$
2. A car is moving with a speed of  $108 \text{ km/hr}$  on a circular path of radius  $500 \text{ m}$ . Its speed is increasing at the rate of  $2 \text{ m/s}$ . What is the acceleration of the car?
  - (a)  $9.8 \text{ m/s}^2$
  - (b)  $2.7 \text{ m/s}^2$
  - (c)  $3.6 \text{ m/s}^2$
  - (d)  $1.8 \text{ m/s}^2$
3. The moment of inertia of uniform circular disc about an axis passing its center is  $6 \text{ kgm}^2$ . its M.I. about an axis perpendicular to its plane and just touching the rim will be
  - (a)  $18 \text{ kg m}^2$
  - (b)  $30 \text{ kg m}^2$
  - (c)  $15 \text{ kg m}^2$
  - (d)  $3 \text{ kg m}^2$
4. A particle undergoes uniform circular motion. About which point on the plane of the circle will the angular momentum of the particle remain conserved?
  - (a) center of the circle
  - (b) on the circumference of the circle
  - (c) inside the circle
  - (d) outside the circle
5. Two particles A and B, initially at rest, moves towards each other under a mutual force of attraction. At the instant when the speed of A is  $u$  and the speed of B is  $2u$ , the speed of center of mass is,
  - (a) Zero

- (b)  $u$
- (c)  $1.5 u$
- (d)  $3 u$

6. The moment of inertia of a body about a given axis is  $1.2 \text{ kg metre}^2$ . Initially, the body is at rest. In order to produce a rotating kinetic energy of  $1500 \text{ joules}$ , an angular acceleration of  $25 \text{ radian/sec}^2$  must be applied about that axis for a duration of

- (a)  $4 \text{ sec}$
- (b)  $2 \text{ sec}$
- (c)  $8 \text{ sec}$
- (d)  $10 \text{ sec}$

7. Two discs has same mass rotates about the same axes.  $r_1$  and  $r_2$  are densities of two bodies ( $r_1 > r_2$ ) then what is the relation between  $I_1$  and

- (a)  $I_2$ .
- (b)  $I_1 > I_2$
- (c)  $I_1 < I_2$
- (d)  $I_1 = I_2$

None of these

8. The kinetic energy of a body is  $4 \text{ joule}$ , and its moment of inertia is  $2 \text{ kg m}^2$  then angular momentum is

- (a)  $4 \text{ kg m}^2/\text{sec}$
- (b)  $5 \text{ kg m}^2/\text{sec}$
- (c)  $6 \text{ kg m}^2/\text{sec}$
- (d)  $7 \text{ kg m}^2/\text{sec}$

9. A mass is revolving in a circle which is in the plane of the paper. The direction of angular acceleration is

- (a) Upward to the radius
- (b) Towards the radius
- (c) Tangential
- (d) At right angle to angular velocity

10. By keeping moment of inertia of a body constant, if we double the time period, then angular momentum of body

- (a) Remains constant

- (b) Becomes half
- (c) Doubles
- (d) Quadruples

### Very Short Question:

1. Can the geometrical centre and C.M. of a body coincide? Give examples.
2. How does the M.I. change with the speed of rotation?
3. Under what conditions, the torque due to an applied force is zero?
4. Is it correct to say that the C.M. of a system of n-particles is always given by average position vectors of the constituent particles? If not, when the statement is true?
5. A cat is able to land on her feet after a fall. Which principle of Physics is being used by her?
6. What is conserved when a planet revolves around a star?
7. If no external torque acts on a body, will its angular velocity remain conserved?
8. A body is rotating at a steady rate. Is a torque acting on the body?
9. What is the other name for angular momentum?
10. Out of two spheres of equal masses, one rolls down a smooth inclined plane of height  $h$  and the other is falling freely through height  $h$ . In which case, the work done is more?

### Short Questions:

1. What is the difference between the centre of gravity and C.M.?
2. There are two spheres of the same mass and radius, one is solid, and the other is hollow. Which of them has a larger moment of inertia about its diameter?
3. What shall be the effect on the length of the day if the polar ice caps of Earth melt?
4. If only an external force can change the state of motion of the C.M. of a body, how does it happen that the internal force of brakes can bring a vehicle to rest?
5. What do you understand by a rigid body?
6. What do you understand by a rigid body?
7. Two equal and opposite forces act on a rigid body. Under what conditions will the body (a) rotate, (Z) not rotate?
8. (a) Why is it easier to balance a bicycle in motion?



(b) Why spokes are fitted in the cycle wheel?

### Long Questions:

1. Discuss the rolling of a cylinder (without slipping) down a rough inclined plane and obtain an expression for the necessary coefficient of friction between the cylinder and the surface.
2. Prove that
  - (a)  $\Delta\omega = \tau \Delta\theta$
  - (b)  $P = \tau \omega$ .

### Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
  - (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
  - (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
  - (c) Assertion is correct, reason is incorrect
  - (d) Assertion is incorrect, reason is correct.

**Assertion:** The Centre of mass of a body may lie where there is no mass.

**Reason:** Centre of mass of body is a point, where the whole mass of the body is supposed to be concentrated.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
  - (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
  - (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
  - (c) Assertion is correct, reason is incorrect
  - (d) Assertion is incorrect, reason is correct.

**Assertion:** The earth is slowing down and as a result the moon is coming nearer to it.

**Reason:** The angular momentum of the earth moon system is conserved.

### Case Study Questions:

1. The cross product of two vectors is given by Vector  $C = A \times B$ . The magnitude of the vector defined from cross product of two vectors is equal to product of magnitudes of the vectors and sine of angle between the vectors. Direction of the vectors is given by right hand corkscrew rule and is perpendicular to the plane containing the vectors.

$$\therefore |\text{vector } C| = AB \sin \theta \text{ and Vector } C = AB \sin \theta \hat{n}$$

Where,  $\hat{n}$  is the unit vector perpendicular to the plane containing the vectors  $A$  and  $B$ . Following are properties of vector product

- a) Cross product does not obey commutative law. But its magnitude obeys commutative law.

$$\begin{aligned} \vec{A} \times \vec{B} &\neq \vec{B} \times \vec{A} \Rightarrow (\vec{A} \times \vec{B}) \\ &= -(\vec{B} \times \vec{A}), |\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| \end{aligned}$$

- c) It obeys distributive law

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

- d) The magnitude cross product of two vectors which are parallel is zero. Since  $\theta = 0$ .  
vector  $|A \times B| = AB \sin 0^\circ = 0$

- e) For perpendicular vectors,  $\theta = 90^\circ$ , vector  $|A \times B| = AB \sin 90^\circ |\hat{n}| = AB$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = \hat{k}; \quad \hat{j} \times \hat{k} = \hat{i}; \quad \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -(\hat{i} \times \hat{j}) = -\hat{k}; \quad \hat{k} \times \hat{j} = -(\hat{j} \times \hat{k}) = -\hat{i}; \quad \hat{i} \times \hat{k} = -(\hat{k} \times \hat{i}) = -\hat{j}$$

- f) The expression for  $a \times b$  can be put in a determinant form which is easy to remember

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

- i. If  $\theta$  is angle between two vectors, then resultant vector is maximum when  $\theta$  is
- 0
  - 90
  - 180
  - None of these

- ii. Cross product is operation performed between
    - a) Two scalar numbers
    - b) One scalar other vector
    - c) 2 vectors
    - d) None of these
  - iii. Define cross product of two vectors
  - iv. State right hand screw rule for finding out direction of resultant after cross product of two vectors.
  - v. Give properties of cross product of parallel vector.
2. Radius of gyration: The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

the moment of inertia of a rigid body analogous to mass in linear motion and depends on the mass of the body, its shape and size, distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation.

Theorem of perpendicular axes

It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body. If we consider a planar body, An axis perpendicular to the body through a point O is taken as the z-axis. Two mutually perpendicular axes lying in the plane of the body and concurrent with z-axis, i.e., passing through O, are taken as the x and y-axes. The theorem states that

$$I_z = I_x + I_y.$$

Theorem of parallel axes

The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

$z$  and  $z'$  are two parallel axes, separated by a distance  $a$ . The  $z$ -axis passes through the centre of mass  $O$  of the rigid body. Then according to the theorem of parallel axes

$$I_{z'} = I_z + Ma^2$$

Where  $I_z$  and  $I_{z'}$  are the moments of inertia of the body about the  $z$  and  $z'$  axes respectively,  $M$  is the total mass of the body and  $a$  is the perpendicular distance between the two parallel axes.

- i. SI unit of radius of gyration
  - a) Metre (m)
  - b)  $M^2$
  - c)  $M^3$
  - d) None of these
- ii. Moment of inertia is analogous to
  - a) Mass
  - b) Area
  - c) Force
  - d) None of these
- iii. Define radius of gyration
- iv. State Theorem of perpendicular axes
- v. State Theorem of parallel axes

✓ Answer Key:

### Multiple Choice Answers-

1. Answer: (b)  $L/4$
2. Answer: (b)  $2.7 \text{ m/s}^2$
3. Answer: (a)  $18 \text{ kg m}^2$
4. Answer: (a) center of the circle
5. Answer: (a) Zero
6. Answer: (b) 2 sec
7. Answer: (b)  $I_1 > I_2$
8. Answer: (a)  $4 \text{ kg m}^2/\text{sec}$
9. Answer: (c) Tangential
10. Answer: (b) Becomes half

### Very Short Answers:

1. Answer: Yes, C.M. and geometrical centre may coincide when the body has a uniform mass density, e.g. C.M. and geometrical centre are the same in case of a sphere, cube and cylinder etc.
2. Answer: M.I. is not affected by the speed of rotation of the body.
3. Answer:

We know that  $\tau = rF \sin \theta$ . If  $\theta = 0$  or  $180$ ,

or

$r = 0$ , then  $\tau = 0$ ,  $r = 0$  means the applied force passes through the axis of rotation.

4. Answer: No, this statement is true when all the particles of the system are of the same mass.
5. Answer: Principle of conservation of angular momentum.
6. Answer: Angular momentum.
7. Answer: No, it is the angular momentum that will be conserved.
8. Answer: No, torque is required only for producing angular acceleration.
9. Answer: Moment of momentum.
10. Answer: Moment of momentum.

### Short Questions Answers:

1. Answer: C.G.: It is the point where the whole of the weight of the body is supposed to be concentrated i.e. on this point, the resultant of the gravitational force on all the particles of the body acts.  
C.M.: It is the point where the whole of the mass of the body may be supposed to be concentrated to describe its motion as a particle.
2. Answer: The hollow sphere shall have greater M.I., as its entire mass is concentrated at the boundary of the sphere which is at maximum distance from the axis.
3. Answer: Melting of polar ice caps will produce water spread around the Earth going farther away from the axis of rotation that will increase the radius of gyration and hence M.I. In order to conserve angular momentum, the angular velocity  $\omega$  shall decrease. So the length of the day  $\left(T = \frac{2\pi}{\omega}\right)$  shall increase.
4. Answer: The internal force of brakes converts the rolling friction into sliding friction. When brakes are applied, wheels stop rotating. When they slide, the force of friction comes into play and stops the vehicle. It is an external force.
5. Answer: A rigid body is that in which the distance between all the constituting particles remains fixed under the influence of external force. A rigid body thus conserves its shape during its motion.
6. Answer:
  - The mutual forces between the particles of a system are called internal forces.



- The forces exerted by some external source on the particles of the system are called external forces.
7. Answer: Two equal and opposite forces acting on a rigid body such that their lines of action don't coincide constitute a couple. This couple produces a turning effect on the body. Hence the rigid body will rotate. If the two equal and opposite forces act in such a way that their lines of action coincide, then the body will not rotate.
8. Answer:
- (a) The rotating wheels of a bicycle possess angular momentum. In the absence of an external torque, neither the magnitude nor the direction of angular momentum can change. The direction of angular momentum is along the axis of the wheel. So the bicycle does not get tilted.
- (b) The cycle wheel is constructed in such a way so as to increase the M.I. of the wheel with minimum possible mass, which can be achieved by using spokes and the M.I. is increased to ensure the uniform speed.

### Long Questions Answers:

1. Answer:

Consider a solid cylinder of mass  $m$ , radius  $R$  and M.I.  $I$  rolling down an inclined plane without slipping as shown in the figure. The condition of rolling down without slipping means that at each instant of time, the point of contact  $P$  of the cylinder with the inclined plane is momentarily at rest and the cylinder is rotating about that as the axis.

Let  $\theta$  = angle of inclination of the plane. The forces acting on the cylinder are:

- The weight  $mg$  of the cylinder acting vertically downward.
- The force of friction  $F$  between the cylinder and the surface of the inclined plane and acts opposite to the direction of motion.
- The normal reaction  $N$  due to the inclined plane acting normally to the plane at the point of contact. The weight  $W$  of the cylinder can be resolved into two rectangular components:
  - (a)  $mg \cos \theta$  along  $\perp$  to the inclined plane.
  - (b)  $mg \sin \theta$  along the inclined plane and in the downward direction. It makes the body move downward.

Let  $a$  = linear acceleration produced in the cylinder,

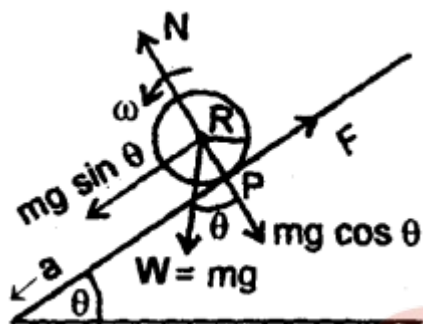
Then according to Newton's 2nd law of motion,

$$ma = mg \sin \theta - F \dots (1)$$

$$\text{and } N = mg \cos \theta \dots (2)$$

If  $\alpha$  = angular acceleration of the cylinder about the axis of rotation, then

$$\tau = I \alpha \dots (3)$$



Here,  $\tau$  is provided by  $F$  i.e.

$$\tau = F.R \dots (4)$$

$\therefore$  from (3) and (4), we get

$$I \alpha = FR$$

or

$$F = \frac{I\alpha}{R} = \frac{I}{R} \cdot \frac{a}{R} \quad (\because a = R\alpha)$$

$$= \frac{Ia}{R^2} \dots (5)$$

$\therefore$  from (1) and (5), we get

$$ma = mg \sin \theta - \frac{Ia}{R^2}$$

or

$$ma + \frac{Ia}{R^2} = mg \sin \theta$$

or

$$a \left( m + \frac{I}{R^2} \right) = mg \sin \theta$$

$$\text{or } a = \frac{mg \sin \theta}{m \left( 1 + \frac{I}{mR^2} \right)}$$

$$\text{or } a = \frac{g \sin \theta}{1 + \frac{I}{mR^2}} \quad \dots (6)$$

$$\text{For solid cylinder, } I = \frac{1}{2} mR^2$$

$$\therefore a = \frac{g \sin \theta}{\left( 1 + \frac{1}{2} \right)} = \frac{2}{3} g \sin \theta$$

$\therefore$  From (5) and (6), we get

$$F = \frac{I}{R^2} \cdot \frac{g \sin \theta}{\left( 1 + \frac{I}{mR^2} \right)}$$

$$F = \frac{mg \sin \theta}{\left( 1 + \frac{mR^2}{I} \right)} \quad \dots (7)$$

$$= \frac{mg \sin \theta}{(1+2)} \quad \left( \because I = \frac{1}{2} mR^2 \right)$$

$$F = \frac{1}{3} mg \sin \theta \quad \dots (8)$$

If  $\mu_s$  be the coefficient of static friction between the cylinder and the surface,

Then

$$\mu_s = \frac{F}{N} = \frac{\frac{1}{3} mg \sin \theta}{mg \cos \theta}$$

$$= \frac{1}{3} \tan \theta$$

For rolling without slipping

$$\frac{F}{N} \leq \mu_s$$

$$\text{or } \frac{1}{3} \tan \theta < \mu_s \quad \dots (9)$$

equation (9) is the required condition for rolling without slipping i.e.,  $\frac{1}{3} \tan \theta$  should be less than equal to  $\mu_s$  i.e., the maximum allowed inclination of the plane with the horizontal is given by

$$\theta_{\max} = \tan^{-1} (3 \mu_s)$$

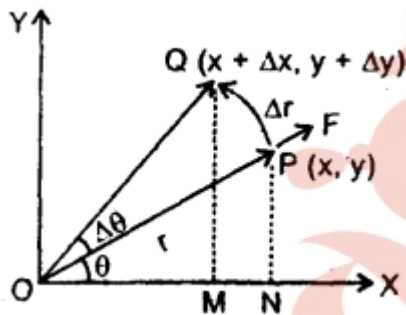
2. Answer:

(a)  $\Delta\omega = \tau \Delta\theta$

Let  $F$  = force applied on a body moving in  $XY$  plane.

$\Delta r$  = linear displacement produced in the body by the force  $F$  in moving from  $P$  to  $Q$ .

If  $\Delta W$  is the small work done by the force, then by definition of work.



$$\Delta W = F \cdot \Delta r \dots (1)$$

In component form,

$$\mathbf{F} = F_x \hat{i} + F_y \hat{j}$$

and  $\Delta \mathbf{r} = \Delta x \hat{i} + \Delta y \hat{j} \dots (2)$

$\therefore$  from (1) and (2), we get

$$\begin{aligned} \Delta W &= (F_x \hat{i} + F_y \hat{j}) \cdot (\Delta x \hat{i} + \Delta y \hat{j}) \\ &= F_x \Delta x + F_y \Delta y \dots (3) \end{aligned}$$

Let  $PN \perp$  on  $X$ -axis &  $\angle PON = \theta$

$\therefore$  in rt  $\angle$   $\Delta PNO$ ,

$$\sin \theta = \frac{y}{r} \dots (4)$$

and  $\cos \theta = \frac{x}{r} \dots (5)$

Also in  $\Delta QMO$ ,

$$x + \Delta x = r \cos (\theta + \Delta\theta)$$

and  $y + \Delta y = r \sin (\theta + \Delta\theta)$

As  $\Delta\theta$  is very small, i.e.  $\Delta\theta \rightarrow 0$ ,  $\cos \Delta\theta \rightarrow 1$  and  $\sin \Delta\theta \rightarrow \Delta\theta$

$$\begin{aligned}\therefore x + \Delta x &= r (\cos \theta \cos \Delta\theta - \sin \theta \sin \Delta\theta) \\ &= r (\cos \theta \cdot 1 - \sin \theta \cdot \Delta\theta) \\ &= x - y \Delta\theta\end{aligned}$$

$$\text{or } \Delta x = -y \Delta\theta \quad \dots (6)$$

$$\begin{aligned}\text{and } y + \Delta y &= r (\sin \theta \cos \Delta\theta + \cos \theta \sin \Delta\theta) \\ &= r (\sin \theta \cdot 1 + \cos \theta \cdot \Delta\theta) \\ &= y + x \Delta\theta\end{aligned}$$

$$\text{or } \Delta y = x \Delta\theta \quad \dots (7)$$

$\therefore$  from (3), (6) and (7), we get

$$\begin{aligned}\Delta\omega &= F_x (-y \Delta\theta) + F_y (x \Delta\theta) \\ &= (x F_y - y F_x) \Delta\theta \\ &= \tau \Delta\theta\end{aligned}$$

(b)  $P = \tau \omega$ .

$$\text{We know that } P = \frac{\Delta\omega}{\Delta t} = \frac{\tau \Delta\theta}{\Delta t} = \tau \frac{\Delta\theta}{\Delta t} = \tau \omega$$

$$\text{where } \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega \text{ if } \Delta t \rightarrow 0$$

$$\therefore P = \tau \omega.$$

### Assertion Reason Answer:

- (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

#### Explanation

As the concept of Centre of mass is only theoretical, therefore in practice no mass may lie at the Centre of mass. For example, Centre of mass of a uniform circular ring is at the Centre of the ring where there is no mass.

- (d) The earth is not slowing down. The angular momentum of the earth – moon system is conserved.

#### Explanation:

The earth is not slowing down. The angular momentum of the earth – moon system is conserved.

### Case Study Answer:

#### 1. Answer

- (a) 0



- ii. (c) 2 vectors
- iii. The cross product of two vectors is given by Vector  $C = A \times B$ . The magnitude of the vector defined from cross product of two vectors is equal to product of magnitudes of the vectors and sine of angle between the vectors.  
 $\therefore |\text{vector } C| = AB\sin\theta$  and **Vector  $C = AB\sin\theta \mathbf{n}$** . Where, cap n is the unit vector perpendicular to the plane containing the vectors A and B.
- iv. We can find the direction of the unit vector with the help of the right-hand rule. In this rule, we can stretch our right hand so that the index finger of the right hand in the direction of the first vector and the middle finger is in the direction of the second vector. Then, the thumb of the right hand indicates the direction of unit vector n.
- v. The cross product of two vectors is zero vectors if both the vectors are parallel or opposite to each other. Conversely, if two vectors are parallel or opposite to each other, then their product is a zero vector. Two vectors have the same sense of direction.  $\theta = 90^\circ$  As we know,  $\sin 0^\circ = 0$  and  $\sin 90^\circ = 1$

$$\vec{X} \times \vec{Y} = |\vec{X}| \cdot |\vec{Y}| \sin\theta$$

$$\vec{X} \times \vec{Y} = |\vec{X}| \cdot |\vec{Y}| \sin 0^\circ$$

$$\vec{X} \times \vec{Y} = |\vec{X}| \cdot |\vec{Y}| \times 0$$

Hence, the cross product of the parallel vectors becomes

$$\vec{X} \times \vec{Y} = 0, \text{ which is a unit vector.}$$

## 2. Answer

- i. (a) Metre (m)
- ii. (c) Mass
- iii. **Radius of gyration:** The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.
- iv. **Theorem of perpendicular axes**  
 It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane

of the body. If we consider a planar body, an axis perpendicular to the body through a point O is taken as the z-axis. Two mutually perpendicular axes lying in the plane of the body and concurrent with z-axis, i.e., passing through O, are taken as the x and y-axes. The theorem states that

$$I_z = I_x + I_y$$

- v. The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes. z and z' are two parallel axes, separated by a distance a. The z-axis passes through the centre of mass O of the rigid body. Then according to the theorem of parallel axes

$$I_{z'} = I_z + Ma^2$$

Where  $I_z$  and  $I_{z'}$  are the moments of inertia of the body about the z and z' axes respectively, M is the total mass of the body and a is the perpendicular distance between the two parallel axes.



# Fukey Education