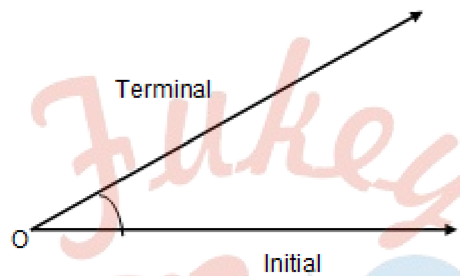
MATHEMATICS

Chapter 3: TRIGONOMETRIC FUNCTION

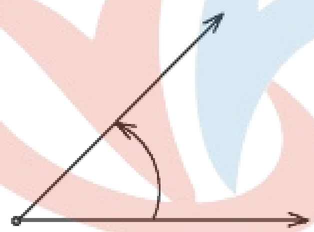
TRIGONOMETRIC FUNCTION

Top Concepts

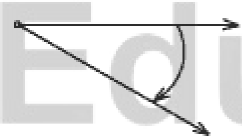
1. An angle is a measure of rotation of a given ray about its initial point. The original position of the ray before rotation is called the initial side of the angle, and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex.



2. If the direction of rotation is anticlockwise, then the angle is said to be positive, and if the direction ofrotation is clockwise, then the angle is negative.

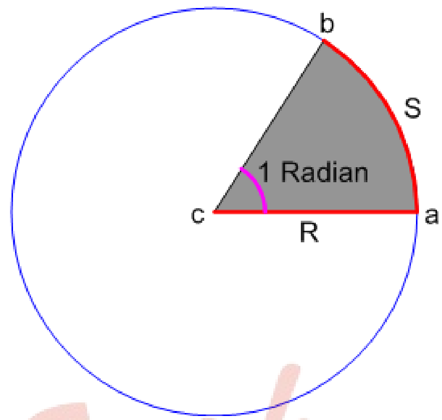


Positive angle—Anticlockwise



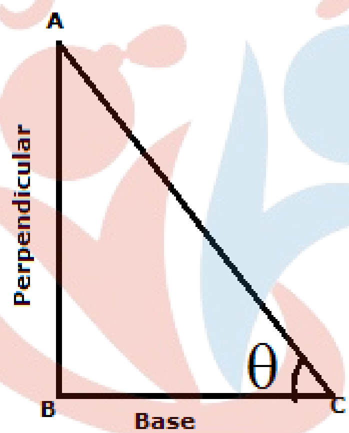
Negative angle—Clockwise

3. If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, then the angle is said to have a measure of one degree. It is denoted by 1°.
4. A degree is divided into 60 minutes, and a minute is divided into 60 seconds. One sixtieth of a degree is called a minute and is written as 1’, and one sixtieth of a minute is called a Second and is written as1”.
Thus, 1° = 60’ and 1’ = 60”.
5. The angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian.



6. Basic trigonometric ratios

Consider the following triangle:



i. Sineθ = $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$

ii. Cosineθ = $\frac{\text{Base}}{\text{Hypotenuse}}$

iii. Tangentθ = $\frac{\text{Perpendicular}}{\text{Base}}$

iv. Cosecantθ = $\frac{\text{Hypotenuse}}{\text{Perpendicular}}$

v. Secantθ = $\frac{\text{Hypotenuse}}{\text{Base}}$

vi. Cotangentθ = $\frac{\text{Base}}{\text{Perpendicular}}$

7. Some trigonometric identities

i. $\sin\theta = \dfrac{1}{\operatorname{cosec}}$ or $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

ii. $\cos\theta = \dfrac{1}{\sec\theta}$ or $\sec\theta = \dfrac{1}{\cos\theta}$

iii. $\tan\theta = \dfrac{1}{\cot}$ or $\cot\theta = \dfrac{1}{\tan\theta}$

iv. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

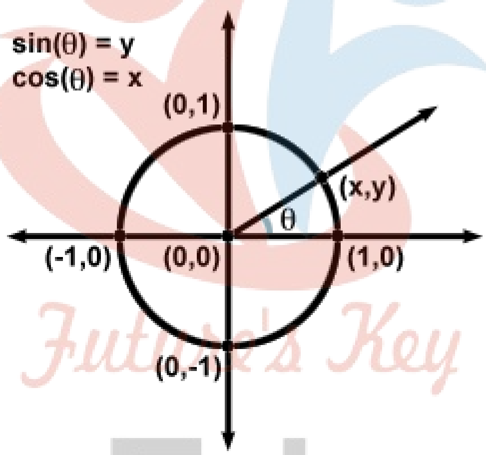
v. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

vi. $\sin^2\theta + \cos^2\theta = 1$

vii. $1 + \tan^2\theta = \sec\ \theta$

viii. $1 + \cot^2\theta = \operatorname{cosec}\ \theta$

8. If a point on the unit circle is on the terminal side of an angle in the standard position, then the sine of such an angle is simply the y-coordinate of the point and the cosine of the angle is the x-coordinate of that point.



9. All the angles which are integral multiples of $\frac{\pi}{2}$ are called quadrantal angles. Values of quadrantal angles are:

$\cos 0 = 1, \sin 0 = 0$

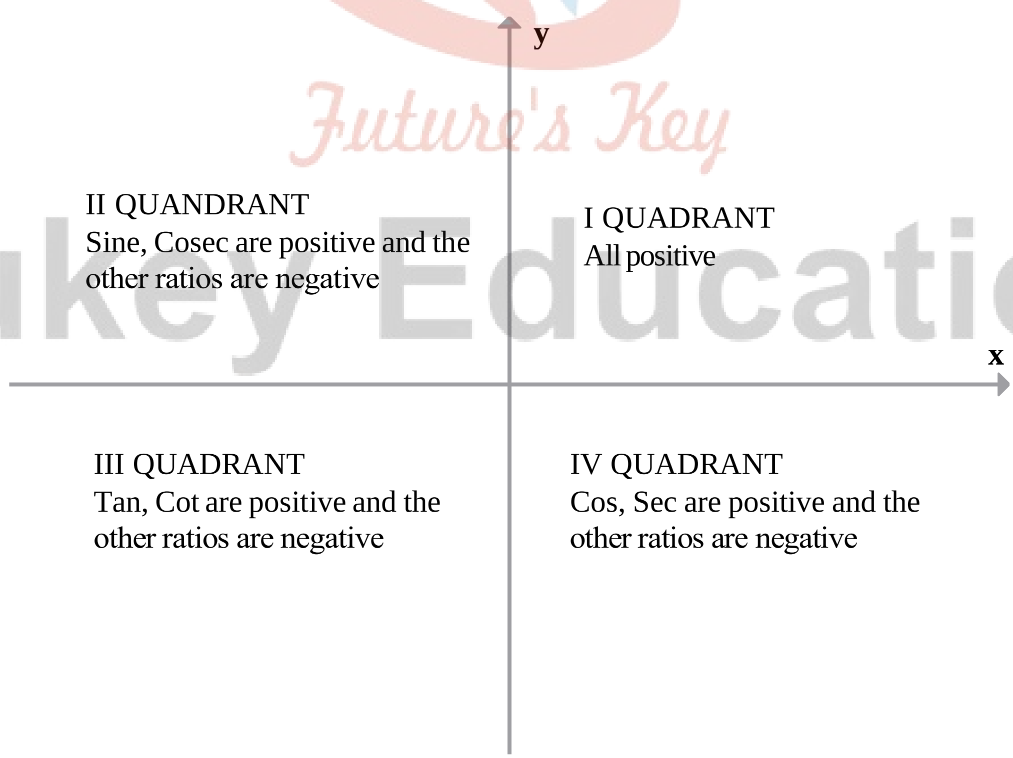
$\cos \frac{\pi}{2} = 0, \sin \frac{\pi}{2} = 1$

$\cos \pi = -1, \sin \pi = 0$

$\cos \frac{3\pi}{2} = 0, \sin \frac{3\pi}{2} = -1$

$\cos 2\pi = 1, \sin 2\pi = 0$

10. Even function: A function f(x) is said to be an even function if f(-x) = f(x) for all x in its domain.
11. Odd function: A function f(x) is said to be an odd function if f(-x) = -f(x) for all x in its domain.
12. Cosine is even and sine is an odd function
    $\cos(-x) = \cos x$
    $\sin(-x) = -\sin x$
13. Signs of trigonometric functions in various quadrants
    In quadrant I, all the trigonometric functions are positive.
    In quadrant II, only sine is positive. In quadrant III, only tan is positive. In quadrant IV, only cosine function is positive. This is depicted as follows



14. In quadrants where Y-axis is positive (i.e. I and II), sine is positive, and in quadrants where X-axis ispositive (i.e. I and IV), cosine is positive.
15. A simple rule to remember the signs of trigonometrical ratios in all the four quadrants is the four letter phrase—**A**ll **S**chool **T**o **C**ollege.
16. A function f is said to be a periodic function if there exists a real number T>0 such that f(x + T) = f(x) for all x. This T is the period of function.
17. Trigonometric ratios of complementary angles
    i. $\sin(90^o - \theta) = \cos\theta$
    ii. $\cos(90^o - \theta) = \sin\theta$
    iii. $ten(90^o - \theta) = \cot\theta$
    iv. $cosec(90^o - \theta) = \sec\theta$
    v. $\sec(90^o - \theta) = cosec\theta$
    vi. $\cot(90^o - \theta) = \tan\theta$
18. Trigonometric ratios of $(90^o + \theta)$ in terms of θ
    i. $\sin(90^o + \theta) = \cos\theta$
    ii. $\cos(90^o + \theta) = -\sin\theta$
    iii. $ten(90^o + \theta) = -\cot\theta$
    iv. $cosec(90^o + \theta) = \sin\theta$
    v. $\sec(90^o + \theta) = cosec\theta$
    vi. $\cot(90^o + \theta) = -\tan\theta$
19. Trigonometric ratios of $(180^o - \theta)$ in terms of θ
    i. $\sin(180^o - \theta) = \sin\theta$
    ii. $\cos(180^o - \theta) = -\cos\theta$
    iii. $ten(180^o - \theta) = -\tan\theta$
    iv. $cosec(180^o - \theta) = cosec\theta$
    v. $\sec(180^o - \theta) = -\sec\theta$
    vi. $\cot(180^o - \theta) = -\cot\theta$
20. Trigonometric ratios of $(180^o + \theta)$ in terms of θ
    i. $\sin(180^o + \theta) = -\sin\theta$
    ii. $\cos(180^o + \theta) = -\cos\theta$
    iii. $ten(180^o + \theta) = \tan\theta$
    iv. $cosec(180^o + \theta) = -cosec\theta$
    v. $\sec(180^o + \theta) = -\sec\theta$
    vi. $\cot(180^o + \theta) = \cot\theta$
21. Trigonometric ratios of $(360^o - \theta)$ in terms of θ
    i. $\sin(360^o - \theta) = -\sin\theta$
    ii. $\cos(360^o - \theta) = \cos\theta$

iii. $ten(360^o - \theta) = -tan\theta$
iv. $cosec(360^o - \theta) = -cosec\theta$
v. $sec(360^o - \theta) = sec\theta$
vi. $cot(360^o - \theta) = -cot\theta$

22. Trigonometric ratios of $(360^o + \theta)$ in terms of θ
i. $sin(360^o + \theta) = sin\theta$
ii. $cos(360^o + \theta) = cos\theta$
iii. $ten(360^o + \theta) = tan\theta$
iv. $cosec(360^o + \theta) =-cosec\theta$
v. $sec(360^o + \theta) = sec\theta$
vi. $cot(360^o + \theta) = cot\theta$

23. In the case of sin $(2\pi + x)$ = sin x, so the period of sine is 2π. Period of its reciprocal is also 2π.
24. In the case of cos $(2\pi + x)$ = cos x, so the period of cos is 2π. Period of its reciprocal is also 2π.
25. In the case of tan $(\pi + x)$ = tan x. Period of tangent and cotangent function is π.
26. The graph of cos x can be obtained by shifting the sine function by the factor $\frac{\pi}{2}$.
27. The tan function differs from the previous two functions in two ways
(i) Function tan is not defined at the odd multiples of $\frac{\pi}{2}$.
(ii) The tan function is not bounded.

28.

Function	**Period**
y = sin x	2π
y = sin (ax)	$\frac{2\pi}{a}$
y = cos x	2π
y = cos (ax)	$\frac{2\pi}{a}$
y = cos 3x	$\frac{2\pi}{3}$
y = sin 5x	$\frac{2\pi}{5}$

29. For a function of the form
$y= kf(ax + b)$
the range will be k times the range of function x, where k is any real number.

If f(x) = sine or cosine function, the range will be equal to R−[−k, k].
If the function is of the form sec x or cosec x, the period is equal to the period of function f by a.
The position of the graph is b units to the right/left of y = f(x) depending on whether b>0 or b<0.

30. The solutions of a trigonometric equation, for which $0 \leq x \leq 2\pi$, are called principal solutions.

29. The expression involving integer 'n' which gives all solutions of a trigonometric equation is called thegeneral solution.

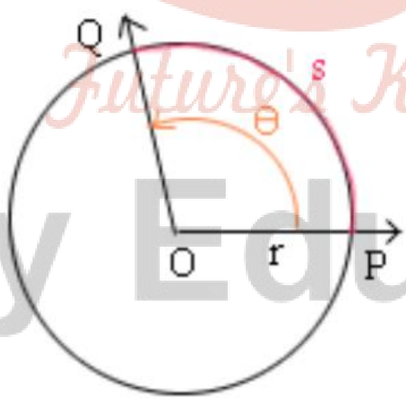
30. The numerical smallest value of the angle (in degree or radian) satisfying a given trigonometric equation is called the Principal Value. If there are two values, one positive and the other negative, which are numerically equal, then the positive value is taken as the Principal Value.

Top Formulae

1. 1 radian = $\frac{180^0}{\pi}$ = 57°16' approximately.

2. 1° = $\frac{180^0}{\pi}$ radians = 0.01746 radians approximately.

3.



s= r θ

Length of arc = radius × angle in radian.

This relation can only be used when θ is in radians.

3. Radian measure = $\frac{\pi}{180}$ × degree measure.

4. degree measure = $\frac{180}{\pi}$ × Radian measure.

5. Values of trigonometric ratios:

	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0
cosec	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not defined	–1	Not defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	–1	Not defined	1
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0	Not defined

6. Domain and range of various trigonometric functions:

Function	**Domain**	**Range**
y = sin x	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	[–1, 1]
y = cos x	$[0, \pi]$	[–1, 1]
y = cosec x	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	R – (–1,1)
y = sec x	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	R – (–1, 1)
y = tan x	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	R
y = cot x	$(0, \pi)$	R

7. Sign convention

	I	II	III	IV
sin x	+	+	—	—
cos x	+	—	—	+
tan x	+	—	+	—
cosec x	+	+	—	—
sec x	+	—	—	+
cot x	+	—	+	—

8. Behavior of Trigonometric Functions in various Quadrants

	I quadrant	**II quadrant**	**III quadrant**	**IV quadrant**
sin	Increases from 0 to 1	Decreases from 1 to 0	Decreases from 0 to —1	Increases from —1 to 0
cos	Decreases from 1 to 0	Decreases from 0 to —1	Increases from —1 to 0	Increases from 0 to 1
tan	Increases from 0 to ∞	Increases from — ∞ to 0	Increases from 0 to ∞	Increases from —∞ to 0
cot	Decreases from ∞ to 0	Decreases from 0 to —∞	Decreases from ∞ to 0	Decreases from 0 to —∞
sec	Increases from 1 to ∞	Increases from —∞ to —1	Decreases from —1 to —∞	Decreases from ∞ to 1
cosec	Decreases from ∞ to 1	Increases from 1 to ∞	Increases from — ∞ to —1	Decreases from — 1 to —∞

9. Basic Formulae

(i) cos (x + y) = cos x cos y — sin x sin y

(ii) cos (x — y) = cos x cos y + sin x sin y

(iii) sin (x + y) = sin x cos y + cos x sin y

(iv) sin (x - y) = sin x cos y - cos x sin y

If none of the angles x, y and (x + y) is an odd multiple of $\frac{\pi}{2}$, then

(v) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

(vi) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

If none of the angles x, y and (x + y) is a multiple of π, then

(vii) $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$

(viii) $\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

10. Allied Angle Relations

$\cos(2\pi - x) = \cos x$ $\quad\quad$ $\sin(2\pi - x) = -\sin x$

$\cos(2n\pi + x) = \cos x$ $\quad\quad$ $\sin(2n\pi + x) = \sin x$

11. Some Important Results

i. $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$

ii. $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x$

iii. $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$

iv. $\sin(x+y+z)$

$= \sin x \cos y \cos z + \cos x \sin y \cos c + \cos x \cos y \sin z - \sin x \sin y \sin z$

v. $\cos(x+y+z)$

$= \cos x \cos y \cos z - \sin x \sin y \cos z - \sin x \cos y \sin z - \cos x \sin y \sin z$

vi. $\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x}$

12. Sum and Difference Formulae

(i) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{}$

(ii) $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{}$

(iii) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{}$

(iv) $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{}$

(v) $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

(vi) $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$

(vii) $2\sin x \cos y = \sin(x+y) + \sin(x-y)$

(viii) $2\cos x \sin y = \sin(x+y) - \sin(x-y)$

13. Multiple Angle Formulae

(i) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$

(ii) $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1+\tan^2 x}$

(iii) $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

(iv) $\sin 3x = 3\sin x - 4\sin^3 x$

(v) $1 + \cos 2x = 2\cos^2 x$

(vi) $1 - \cos 2x = 2\sin^2 x$

(vi) $\cos 3x = 4\cos^3 x - 3\cos x$

(vii) $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

(viii) $\cos x \cdot \cos 2x \cdot \cos 2^2 x \cdot \cos 2^3 x \cdot \ldots \cdot \cos 2^{n-1} x = \frac{\sin 2^n x}{2^n \sin x}$

(ix) Let $x = \frac{\pi}{2^n +}$ then we have $2^n \cos x \cdot \cos 2x \cdot \cos 2^2 x \cdot \cos 2^3 x \cdot \ldots \cdot \cos 2^{n-1} x = 1$

(x) $\sin x \cdot \sin(60^\circ - x) \cdot \sin(60^\circ - x) = \frac{\sin 3x}{4}$

(xi) $\cos x \cdot \cos(60^\circ - x) \cdot \cos(60^\circ - x) = \frac{\cos 3x}{4}$

(xii) $(1 + \sec 2x)(1 + \sec 4x)(1 + \sec 8x)\ldots(1 + \sec 2^n x) = \tan 2^n x \cot x$

14. Trigonometric ratios of angle in terms of half angle

i. $\sin x = 2 \sin \frac{x}{} \cos \frac{x}{2}$

ii. $\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{}$

iii. $\cos x = 2 \cos^2 \frac{x}{2} - 1$

iv. $\cos x = 1 - 2 \sin^2 \frac{x}{2}$

v. $1 + \cos x = 2 \cos^2 \frac{x}{2}$

vi. $1 - \cos x = 2 \sin^2 \frac{x}{2}$

vii. $\tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}$

viii. $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

ix. $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

x. $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{}}$

xi. $\sin \frac{x}{2} = + \sqrt{\frac{1 - \cos x}{}}$

xii. $\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$

15. Trigonometrical ratios of angle in terms of $\frac{x}{3}$ angle:

i. $\sin x = 3\sin\frac{x}{3} - 4\sin^3\frac{x}{3}$

ii. $\cos x = 4\cos^3\frac{x}{3} - 3\cos\frac{x}{3}$

iii. $\tan x = \dfrac{3\tan\frac{x}{3} - \tan^3\frac{x}{3}}{1 - 3\tan^2\frac{x}{3}}$

16. Trigonometrical ratios of important angles:

i. $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$

ii. $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4}$

iii. $\cos 18^\circ = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

iv. $\sin 36^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

17. Trigonometric equations:

No.	Equations	General solution	Principal value
1	$\sin\theta = 0$	$\theta = n\pi, n\in Z$	$\theta = 0$
2	$\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2}, n\in Z$	$\theta = \frac{\pi}{2}$
3	$\tan\theta = 0$	$\theta = n\pi$	$\theta = 0$
4	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n\alpha$ $n\in Z$	$\theta = \alpha$
5	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha\ n\in Z$	$\theta = 2\alpha, \alpha > 0$
6	$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha\ n\in Z$	$\theta = \alpha$
7	$\sin\theta = \sin^2\alpha$	$\theta = n\pi \pm \alpha, n\in Z$	
8	$\cos\theta = \cos^2\alpha$	$\theta = n\pi \pm \alpha, n\in Z$	
9	$\tan\theta = \tan^2\alpha$	$\theta = n\pi \pm \alpha, n\in Z$	

18. The equation $a\cos\theta + b\sin\theta = c$ is solvable for $|c| \leq \sqrt{a^2\ b^2}$.

19. (i) $\sin\theta = k = \sin(n\pi + (-1)^n\alpha), n \in Z$

$\theta = n\pi + (-1)^n \alpha,\ n \in Z$

$\operatorname{cosec}\theta = \operatorname{cosec}\alpha \Rightarrow \sin\theta = \sin\alpha$

$\theta = n\pi + (-1)^n \alpha,\ n \in Z$

(ii) $\cos\theta = k = \cos(2n\pi \pm \alpha),\ n \in$

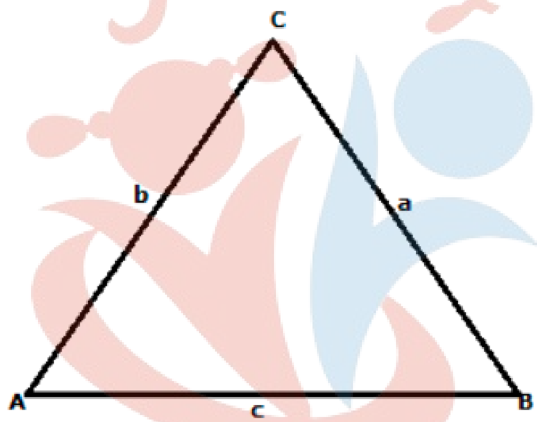
$Z\theta = 2n\pi \pm \alpha,\ n \in Z$

20. Sine Rule: The sine rule states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



21. Law of Cosine

In any ΔABC, $a^2 = b^2 + c^2 - 2bc\cos A, \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$b^2 = c^2 + a^2 - 2ac\cos B, \cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$a^2 = b^2 + c^2 - 2bc\cos A, \cos C = \dfrac{b^2 + c^2 - a^2}{2bc}$

22. Projection Formulae:

(i) a = b cosC + c cosB

(ii) b = c cos A + acosC

(iii) c = acosB b cos A

23. Napier's Analogy (Law of Tangents)

(i) $\tan\left(\frac{B-C}{2}\right)=\left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$

(ii) $\tan\left(\frac{A-B}{2}\right)=\left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$

(iii) $\tan\left(\frac{C-A}{2}\right)=\left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$

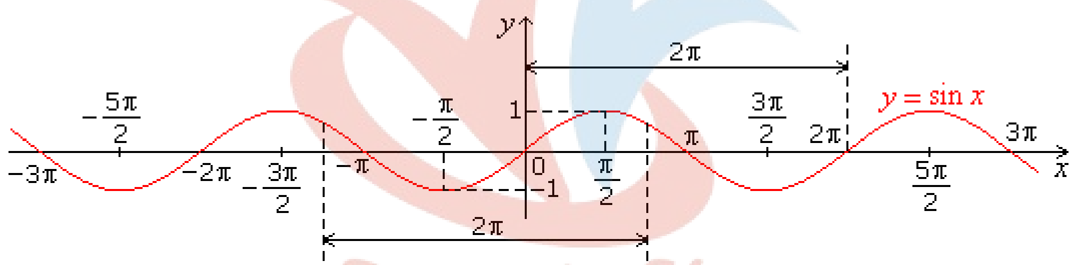
24. Area of ΔABC is given by

$\Delta=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C$

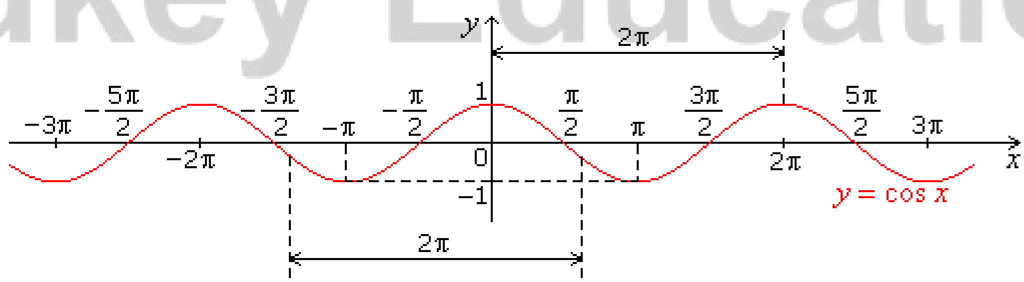
Top diagrams

1. Graphs helps in visualization of properties of trigonometric functions. The graph of y = sinθ can be drawn by plotting a number of points (θ, sin θ) as θ takes a series of different values. Because the sine function is continuous, these points can be joined with a smooth curve. Following similar procedures, graphs of other functions can be obtained.

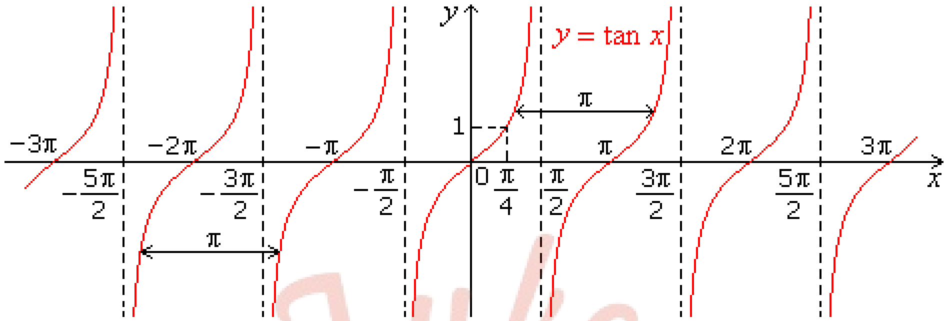
i. Graph of sin x



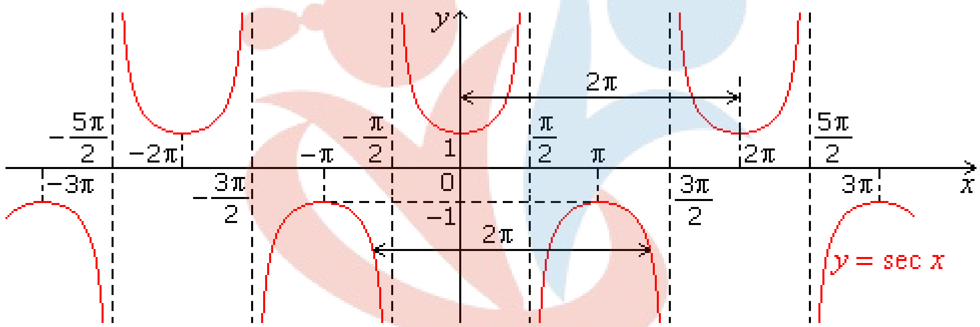
ii. Graph of cos x



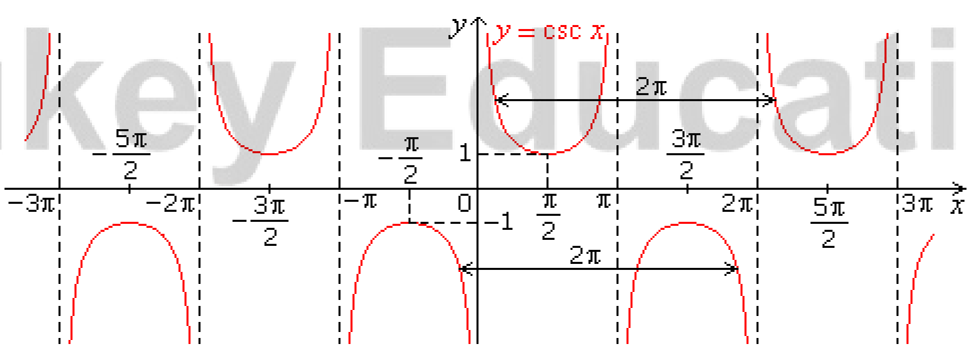
iii. Graph of tan x



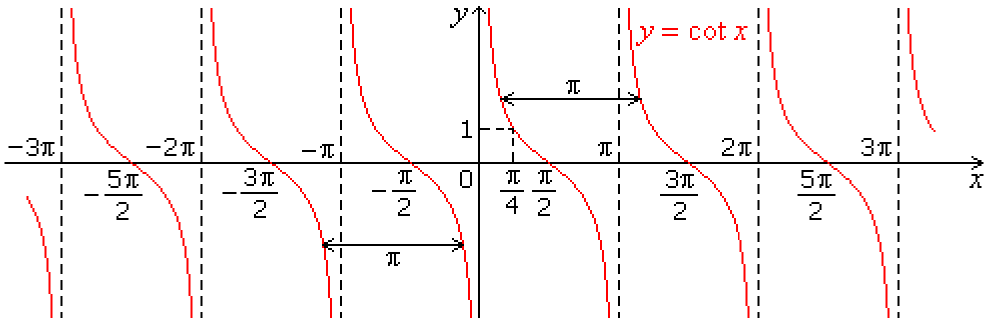
iv. Graph of sec x



v. Graph of cosec x

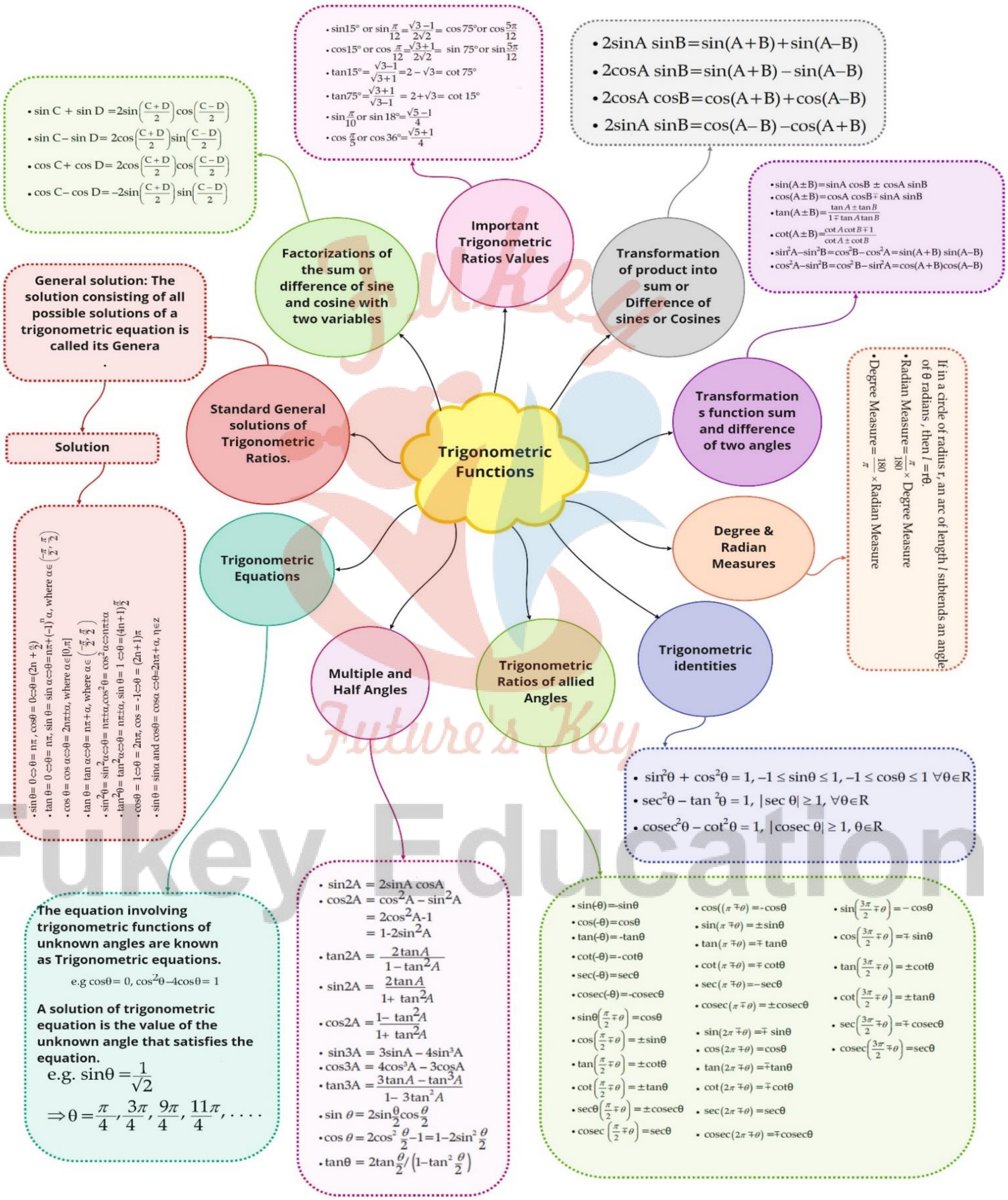


vi. Graph of cot x


y
y = cot x
π
1
−3π
−2π
−π
π
2π
3π
0
x
−5π/2
−3π/2
−π/2
π/4
π/2
3π/2
5π/2
π

Fukey
Future's Key

Class : 11th mathematics
Chapter- 3: Trigonometric Functions



Important Questions

Multiple Choice questions-

Question 1. The value of sin 15 + cos 15 is

(a) 1

(b) 1/2

(c) √3/2

(d) √3

Question 2. The value of tan A/2 – cot A/2 + 2cot A is

(a) 0

(b) 1

(c) -1

(d) None of these

Question 3. The value of 4 × sin x × sin(x + π/3) × sin(x + 2π/3) is

(a) sin x

(b) sin 2x

(c) sin 3x

(d) sin 4x

Question 4. If tan x = (cos 9 + sin 9)/(cos 9 – sin 9), then x =

(a) 45

(b) 54

(c) 36

(d) None of these

Question 5. In a triangle ABC, sin A – cos B = cos C, then angle B is

(a) π/2

(b) π/3

(c) π/4

(d) π/6

Question 6. The value of cos 420° is

(a) 0

(b) 1

(c) 1/2

(d) √3/2

Question 7. If in a triangle ABC, tan A + tan B + tan C = 6 then the value of cot A × cot B × cot C is

(a) 1/2

(b) 1/3

(c) 1/4

(d) 1/6

Question 8. If a × cos x + b × cos x = c, then the value of $(a \times \sin x - b \times \cos x)^2$ is

(a) $a^2 + b^2 + c^2$

(b) $a^2 - b^2 - c^2$

(c) $a^2 - b^2 + c^2$

(d) $a^2 + b^2 - c^2$

Question 9. When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is

(a) 30°

(b) 60°

(c) 75°

(d) 45°

Question 10. In any triangle ABC, if cos A/a = cos B/b = cos C/c and the side a = 2, then the area of the triangle is

(a) √3

(b) √3/4

(c) √3/2

(d) 1/√3

Very Short:

1. Find the radian measure corresponding to 5° 37" 30""
2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$.
3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°
4. Find the value of $\frac{19\pi}{3}$.

5. Find the value of sin(–1125°)
6. Find the value of tan 15°
7. If sin A = $\frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find cos A
8. If tan A = $\frac{a}{a+1}$ and tan B = $\frac{1}{2a+1}$ then find the value of A + B.
9. Express sin 12θ + sin 4θ as the product of sines and cosines.
10. Express 2 cos4x sin2x as an algebraic sum of sines or cosines.

Short Questions:

1. The minute hand of a watch is 1.5 cm long. How far does it tip move in 40 minute?
2. Show that tan 3x. tan 2x. tan x = tan 3x – tan 2x – tan x
3. Find the value of tan $\frac{\pi}{8}$.
4. Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}$
5. If in two circles, arcs of the same length subtend angles 600 and 750 at the center find the ratio of their radii.

Long Questions:

1. If Sin α + Sin β = a and Cos α + Cos β = b show that Cos (α + β) = $\frac{b^2-a^2}{b^2+a^2}$
2. Prove that Cos α + Cos β + Cos γ + Cos (α + β + γ)

   $= 4\text{ Cos}\left(\frac{\alpha+\beta}{2}\right).\text{ Cos}\left(\frac{\beta+\gamma}{2}\right).\text{ Cos}\left(\frac{\gamma+\alpha}{2}\right)$
3. Prove that Sin3x + Sin2x - Sin2x = 4Sinx.cos $\frac{x}{2}$. cos $\frac{3x}{2}$
4. Prove that $2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$
5. Find the value of tan (α + β) Given that.

   $\text{Cot }\alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\text{Sec }\beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$

Assertion Reason Questions:

1. In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

   Assertion (A) : $\sin^{-1}(\sin(2\pi/3)) = 2\pi/3$

   Reason (R) : $\sin^{-1}(\sin\theta) = \theta$, if q ∈ [(-π)/2, π/2]

   (i) Both A and R are true and R is the correct explanation of A

(ii) Both A and R are true but R is NOT the correct explanation of A

(iii) A is true but R is false

(iv) A is false and R is True

2. In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

Assertion (A) : Principal value of $\cos^{-1}(1)$ is π

Reason (R) : Value of cos 0° is 1

(i) Both A and R are true and R is the correct explanation of A

(ii) Both A and R are true but R is NOT the correct explanation of A

(iii) A is true but R is false

(iv) A is false and R is True

Answer Key:

MCQ

1. (c) √3/2

2. (a) 0

3. (c) sin 3x

4. (b) 54

5. (a) π/2

6. (c) 1/2

7. (d) 1/6

8. (d) $a^2 + b^2 - c^2$

9. (d) 45°

10. (a) √3

Very Short Answer:

1. $\left(\frac{\pi}{32}\right)^c$

2. 39°22"30""

3. $\frac{5\pi}{12} cm$

4. $\sqrt{3}$

5. $\frac{-1}{\sqrt{2}}$

6. $2-\sqrt{3}$

7. $\frac{-4}{5}$

8. 45°

9. 2 sin8θ cos4θ

10. sin 6x – sin2x

Short Answer:

1. r = 1.5 cm

Angle made in 60 mint = 360˚

Angle made in 1 min = $\frac{360}{60}$ = 60˚

Angle made in 40 mint = 6 × 40

= 240˚

$\Theta = \frac{l}{r}$

$\cancel{240}^{4} \times \frac{\pi}{\cancel{180}\,3} = \frac{l}{1.5}$

$\frac{4\times 3.14}{\cancel{3}} = \frac{l}{\frac{\cancel{1.5}\,\cancel{5}}{\cancel{10}\,2}}$

$2\times 3.14 = l$

$6.28 = l$

$l = 6.28$ cm

2.

Let 3x = 2x + x

tan 3x = tan (2x + x)

$\frac{\tan\ 3x}{1} = \frac{\tan\ 2x + \tan\ x}{1 - \tan\ 2x.\ \tan\ x}$

$\tan\ 3x\ (1 - \tan\ 2x.\ \tan\ x) = \tan\ 2x + \tan\ x$

$\tan\ 3x - \tan\ 3x.\ \tan\ 2x.\ \tan\ x = \tan\ 2x + \tan\ x$

$\tan\ 3x.\ \tan\ 2x.\ \tan\ x = \tan\ 3x - \tan\ 2x - \tan\ x$

3.

Let $x = \frac{\pi}{8}$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} t$$

$$\tan\left(2\cdot\frac{\pi}{8_4}\right) = \frac{2\tan \pi/8}{1-\tan^2 \pi/8}$$

$$1 = \frac{2\tan \pi/8}{1-\tan^2 \pi/8}$$

put $\tan \pi/8 =$

$$\frac{1}{1} = \frac{2t}{1-t^2}$$

$$2t = 1-t^2$$

$$t^2 + 2t - 1 = 0$$

$$t = \frac{-2 \pm 2\sqrt{2}}{2\times 1}$$

$$= -1 \pm \sqrt{2}$$

$$= \pm\sqrt{2} - 1$$

$$= \sqrt{2}-1 \text{ or } -\sqrt{2}-1$$

$$\tan \pi/8 = \sqrt{2}-1$$

4.

$$\text{L.H.S} = \frac{Sin(x+y)}{Sin(x-y)}$$

$$= \frac{Sinx.Cosy + Cosx.Siny}{Sinx.Cosy - Cosx.Siny}$$

DividingNandDbyCosx.Cosy

$$= \frac{\tan x + \tan y}{\tan x - \tan y}$$

5.

$$\theta = \frac{l}{r_1}$$

$$60 \times \frac{\pi}{18} = \frac{l}{r_1}$$

$$r_1 = \frac{3l}{\pi} \quad (1)$$

$$\theta = \frac{l}{r_2}$$

$$75 \times \frac{\pi}{18} = \frac{l}{r_2}$$

$$r_2 = \frac{12l}{5\pi} \quad (2)$$

(1) ÷ (2)

$$\frac{r_1}{r_2} = \frac{\frac{3l}{\pi}}{\frac{12l}{5\pi}}$$

$$= \frac{3l}{\pi} \times \frac{5\pi}{12l}$$

$$= 5 : 4$$

Long Answer:

1.

$$b^2 + a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 + (\text{Sin } \alpha + \text{Sin } \beta)^2$$

$$= \text{Cos}^2 \alpha + \text{Cos}^2 \beta + 2 \text{ Cos } \alpha . \text{ Cos } \beta + \text{Sin}^2 \alpha + \text{Sin}^2 \beta + 2Sin \alpha . \text{ Sin } \beta$$

$$= 1 + 1 + 2 (\text{Cos } \alpha . \text{ Cos } \beta + \text{Sin } \alpha . \text{ Sin } \beta)$$

$$= 2 + 2 \text{ Cos } (\alpha - \beta) \quad (1)$$

$$b^2 - a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 - (\text{Sin } \alpha + \text{Sin } \beta)^2$$

$$= (\text{Cos}^2 \alpha - \text{Sin}^2 \beta) + (\text{Cos}^2 \beta - \text{Sin}^2 \alpha) + 2 \text{ Cos } (\alpha + \beta)$$

$= \text{Cos}(\alpha + \beta)\ \text{Cos}(\alpha - \beta) + \text{Cos}(\beta + \alpha)\text{Cos}(\alpha - \beta) + 2\ \text{Cos}(\alpha + \beta)$

$= 2\ \text{Cos}(\alpha + \beta).\ \text{Cos}(\alpha - \beta) + 2\ \text{Cos}(\alpha + \beta)$

$= \text{Cos}(\alpha + \beta)\ [2\ \text{Cos}(\alpha - \beta) + 2]$

$= \text{Cos}(\alpha + \beta).\ (b^2 + a^2)$ [from (1)

$\frac{b^2 - a^2}{b^2 + a^2} = \text{Cos}(\alpha + \beta)$

2. L. H. S.

$= \text{Cos}\,\alpha + \text{Cos}\,\beta + \text{Cos}\,\gamma + \text{Cos}(\alpha + \beta + \gamma)$

$= \text{Cos}\,\alpha + \text{Cos}\,\beta) + [\text{Cos}\,\gamma + \text{Cos}(\alpha + \beta + \gamma)]$

$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta+\gamma+\gamma}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+\gamma-\gamma}{2}\right)$

$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)$

$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[\text{Cos}\left(\frac{\alpha-\beta}{2}\right) + \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$

$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right).\ \text{Cos}\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right]$

$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\alpha+\gamma}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right)\right]$

$= 4\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right).\ \text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$

3.

$(\sin 3x - \sin x) + \sin 2x$

$= 2\cos\left(\frac{3x+x}{2}\right).\sin\left(\frac{3x-x}{2}\right) + \sin 2x$

$= 2\cos 2x.\sin x + \sin 2x$

$= 2\cos 2x.\sin x + 2\sin x\cos x$

$= 2\sin x[\cos 2x + \cos x]$

$= 2\sin x\left[2\cos x\frac{3x}{2}.\cos\frac{x}{2}\right]$

$= 4\sin x \cos\frac{3x}{2}\cos\frac{x}{2}$

4. L.H.S.

$= 2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= \cos\left(\frac{\pi}{13} + \frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{13} - \frac{9\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= -\cos\frac{3\pi}{13} - \cos\frac{5\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= 0$

5.

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$ (1)

$\text{Cot}\,\alpha = \frac{1}{2},$

$\Rightarrow \tan\alpha = 2$

$1 + \tan^2\beta = \text{Sec}^2\beta$

$1 + \tan^2\beta = \left(\frac{-5}{3}\right)^2 \quad \left[\because \text{Sec}\,\beta = \frac{-5}{3}1\right)$

$\tan\beta = \pm\frac{4}{3}$

$\tan\beta = -\frac{4}{3} \left[\because \beta \in \left(\frac{\pi}{2}/x\right)\right.$

put $\tan\alpha$, and $\tan\beta$ in eq.

$\tan(\alpha + \beta) = \frac{2 - \frac{4}{3}}{1 - 2\left(-\frac{4}{3}\right)}$

$= \frac{2}{11}$

Assertion Reason Answer:

1. (iv) A is false and R is True

2. (i) Both A and R are true and R is the correct explanation of A