

MATHEMATICS

Chapter 3: Pair of Linear Equations in Two Variables



Pair of Linear Equations in Two Variables

1. A pair of Linear Equations in two variables:

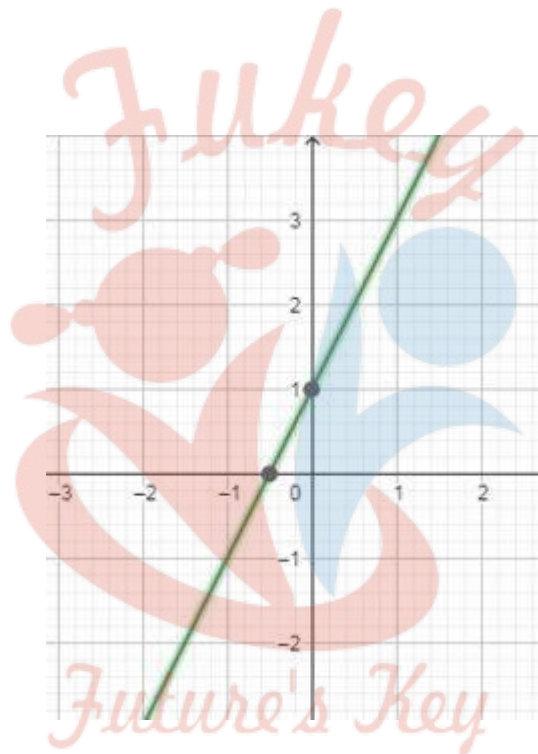
- An equation of the form $ax + by + c = 0$, where a , b and c are real numbers, such that a and b are not both zero, is called a **linear equation in two variables**.
- Two linear equations in same two variables x and y are called **pair of linear equations in two variables**.

Geometrical Representation of a Linear Equation

Geometrically, a linear equation in two variables can be represented as a straight line.

$$2x - y + 1 = 0$$

$$\Rightarrow y = 2x + 1$$



Graph of $y = 2x + 1$

Plotting a Straight Line

The graph of a linear equation in two variables is a straight line. We plot the straight line as follows:

- Take any value for one of the variables ($x_1 = 0$) and substitute it in the equation to get the corresponding value of the other variable (y_1).
- Repeat this again (put $y_2 = 0$, get x_2) to get two pairs of values for the variables which represent two points on the Cartesian plane. Draw a line through the two points.

2. Types of Polynomials based on Degree

Linear Polynomial

A polynomial whose degree is one is called a linear polynomial.

For example, $2x+1$ is a linear polynomial.

(1)

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.

For example, $3x^2 + 8x + 5$ is a quadratic polynomial.

Cubic Polynomial

A polynomial of degree three is called a cubic polynomial.

For example, $2x^3 + 5x^2 + 9x + 15$ is a cubic polynomial.

3. Graph of the polynomial x^n

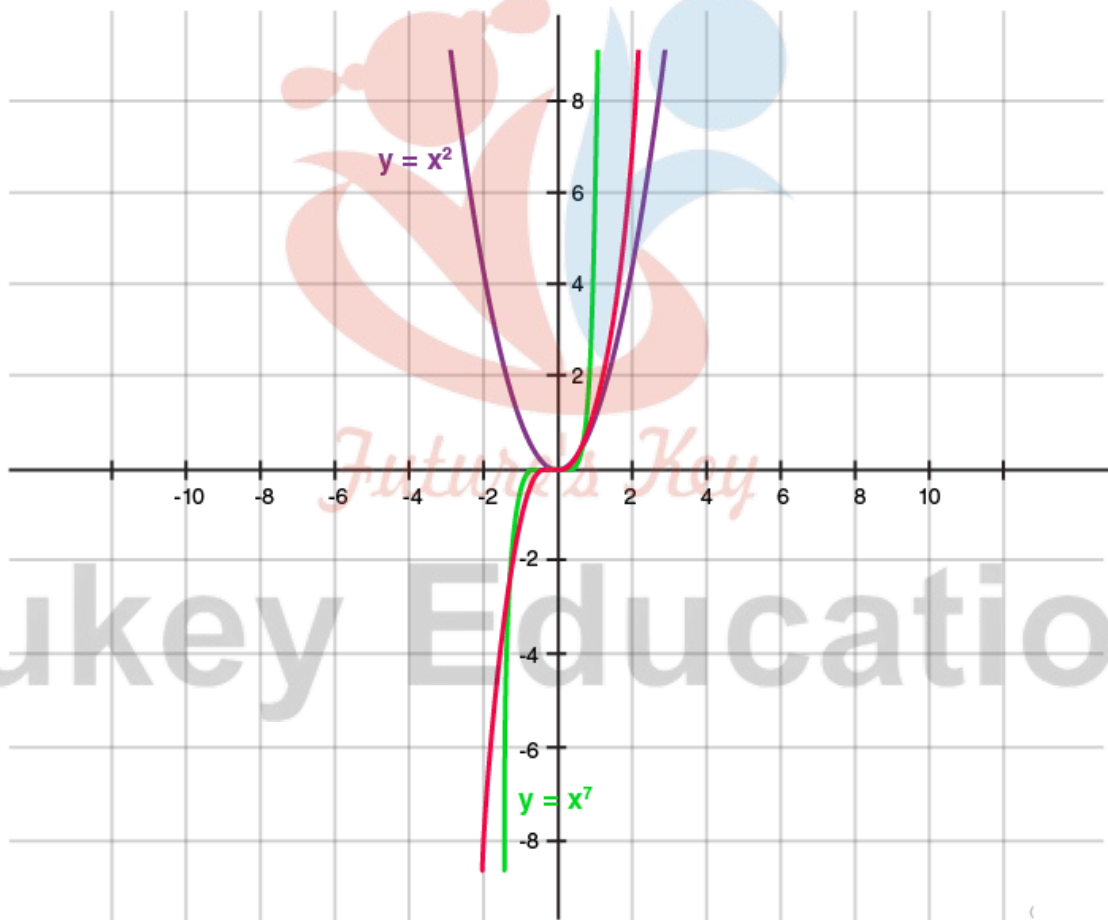
For a polynomial of the form $y = x^n$ where n is a whole number:

as n increases, the graph becomes steeper or draws closer to the Y-axis

If n is odd, the graph lies in the first and third quadrants

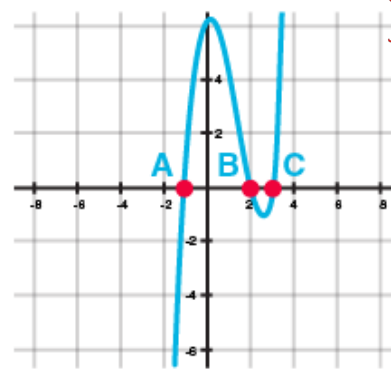
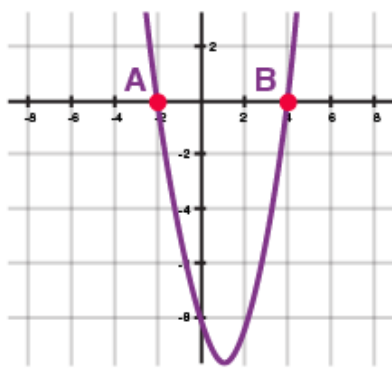
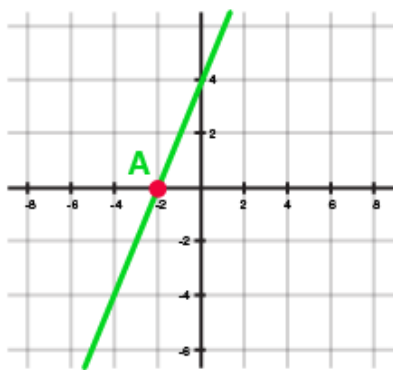
If n is even, the graph lies in the first and second quadrants

The graph of $y = -x^n$ is the reflection of the graph of $y = x^n$ on the x-axis



4. Geometrical Meaning of Zeros of a Polynomial

Geometrically, zeros of a polynomial are the points where its graph cuts the x-axis.



(i) One zero (ii) Two zeros (iii) Three zeros

Here A, B and C correspond to the zeros of the polynomial represented by the graphs.

Number of Zeros

In general, a polynomial of degree n has at most n zeros.

- A linear polynomial has one zero,
- A quadratic polynomial has at most two zeros.
- A cubic polynomial has at most 3 zeros.

5. The **general form** of a pair of linear equations in two variables is

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that

6. A system of linear equations in two variables represents two lines in a plane. For two given lines in a plane there could be three possible cases:

- i. The two lines are intersecting, i. e., they **intersect at one point**.
- ii. The two lines are **parallel**, i.e., they do not intersect at any real point.
- iii. The two lines are **coincident** lines, i.e., one line overlaps the other line.

7. A system of simultaneous linear equations is said to be

- **Consistent**, if it has **at least one solution**.
- **In-consistent**, if it has **no solution**.

8. If the lines

- i. Intersect at a point, then that point gives the **unique solution** of the system of equations. In this case system of equations is said to be **consistent**.
- ii. Coincide (overlap), then the pair of equations will have **infinitely many solutions**. System of equations is said to be **consistent**.
- iii. are parallel, then the pair of equations has **no solution**. In this case pair of

(3)

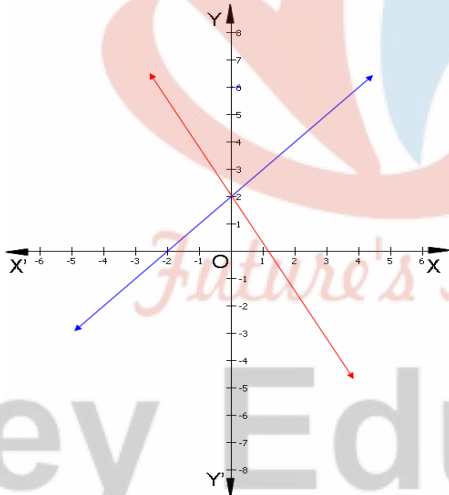
equations is said to be **inconsistent**.

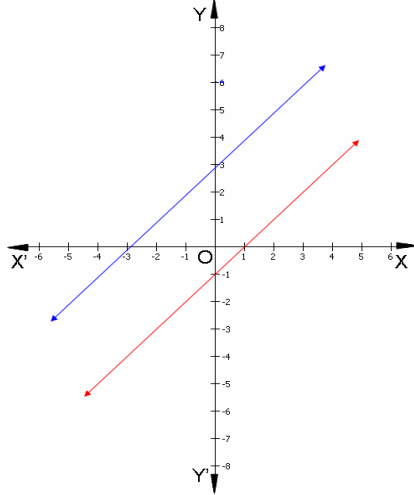
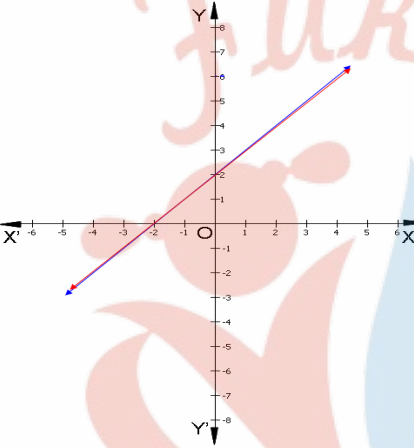
9. Solution of a pair of Linear Equations in two variable:

System of equations can be solved using **Algebraic** and **Graphical Methods**.

10. Graphical Method:

- A linear equation in two variables is represented geometrically by a **straight line**.
- The graph of a pair of linear equations in two variables is represented by two lines.
Steps:
 - i. Draw the graphs of both the equations by finding two solutions for each.
 - ii. Plot the points and draw the lines passing through them to represent the equations.
 - iii. The behaviour of lines representing a pair of linear equations in two variables and the existence of solutions can be summarised as follows:

Ratio of Coefficients	Graphical Representation	Nature of Solution	Defined as
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Lines are intersecting 	Unique solution	Consistent pair of equations
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Lines are parallel	No solution	Inconsistent pair of equations

			
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	<p>Lines are coincident</p> 	Infinitely many solutions	Dependent (consistent) pair of equations

11. Algebraic Method:

The most commonly used **Algebraic Methods** to solve a pair of linear equations in two variables are:

- i. Substitution method
- ii. Elimination method
- iii. Cross-multiplication method

12. Substitution Method:

Steps followed for solving linear equations in two variables, using **substitution method**:

Step 1: Express the value of one variable, say y in terms of other variable x from either equation, whichever is convenient.

Step 2: Substitute the value of y in other equation and reduce it to an equation in one variable, i.e. in terms of x . There will be three possibilities:

- a. If reduced equation is linear in x , then solve it for x to get a **unique solution**.
- b. If reduced equation is a true statement without x , then system has **infinite solutions**.

c. If reduced equation is a false statement without x , then system has **no solution**.

Step 3: Substitute the value of x obtained in step 2, in the equation used in step 1, to obtain the value of y .

Step 4: The values of x and y so obtained is the coordinates of the solution of system of equations.

13. Elimination Method:

Steps followed for solving linear equations in two variables, by **elimination Method**:

Step 1: Multiply both the equations by some suitable non-zero constants to make the coefficients of variable x (or y) equal.

Step 2: Add or subtract both the equations to eliminate the variable whose coefficients are equal.

- If an equation in one variable y (or x) is obtained, solve it for variable y (or x).
- If a true statement involving no variable is obtained then the system has **infinite solutions**.
- If a false statement involving no variable is obtained then the system has **no solution**.

Step 3: Substitute the value of variable y (or x) in either of the equation to get the value of other variable.

14. Cross Multiplication Method:

Steps followed for solving linear equations in two variables, by **cross multiplication method**:

Step 1: Write the equations in the general form.

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Step 2: Arrange these in the following manner.

$$\frac{x}{\begin{array}{c} b_1 \quad c_1 \\ b_2 \quad c_2 \end{array}} = \frac{y}{\begin{array}{c} c_1 \quad a_1 \\ c_2 \quad a_2 \end{array}} = \frac{1}{\begin{array}{c} a_1 \quad b_1 \\ a_2 \quad b_2 \end{array}}$$

Here, the arrows between two numbers (coefficients) mean that they are to be multiplied and the second product is to be subtracted from the first product.

Step 3: Cross multiply:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

a. Comparing (1) and (3), we get the value of x

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

b. Comparing (2) and (3), we get the value of y

$$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

From the above equations, obtain the value of x and y provided $a_1b_2 - a_2b_1 \neq 0$.

15. Equations which are not linear but can be reduced to linear form by some suitable substitutions are called equations reducible to linear form.

Reduced equation can be solved by any of the algebraic method (substitution, elimination or cross multiplication) of solving linear equation.

16. While solving problems based on time, distance and speed; following knowledge may be useful:

If speed of a boat in still water = u km/hr,

Speed of the current = v km/hr Then,

Speed upstream = (u - v) km/hr

Speed downstream = (u + v) km/hr

17. Factorization of Polynomials

Quadratic polynomials can be factorized by splitting the middle term.

For example, consider the polynomial $2x^2 - 5x + 3$

Splitting the middle term:

The middle term in the polynomial $2x^2 - 5x + 3$ is $-5x$. This must be expressed as a sum of two terms such that the product of their coefficients is equal to the product of 2 and 3 (coefficient of x^2 and the constant term)

-5 can be expressed as $(-2) + (-3)$, as $-2 \times -3 = 6 = 2 \times 3$

Thus, $2x^2 - 5x + 3 = 2x^2 - 2x - 3x + 3$

Now, identify the common factors in individual groups

$2x^2 - 2x - 3x + 3 = 2x(x-1) - 3(x-1)$

Taking $(x-1)$ as the common factor, this can be expressed as:

$2x(x-1) - 3(x-1) = (x-1)(2x-3)$

18. Relationship between Zeroes and Coefficients of a Polynomial

For Quadratic Polynomial:

If α and β are the roots of a quadratic polynomial ax^2+bx+c , then,

$$\alpha + \beta = -b/a$$

Sum of zeroes = -coefficient of x / coefficient of x²

$$\alpha\beta = c/a$$

Product of zeroes = constant term / coefficient of x²

For Cubic Polynomial

If α , β and γ are the roots of a cubic polynomial ax^3+bx^2+cx+d , then

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a$$

19. Division Algorithm

To divide one polynomial by another, follow the steps given below.

Step 1: arrange the terms of the dividend and the divisor in the decreasing order of their degrees.

Step 2: To obtain the first term of the quotient, divide the highest degree term of the dividend by the highest degree term of the divisor Then carry out the division process.

Step 3: The remainder from the previous division becomes the dividend for the next step. Repeat this process until the degree of the remainder is less than the degree of the divisor.

$$\begin{array}{r}
 x - 2 \\
 \hline
 -x^2 + x - 1 \overline{) -x^3 + 3x^2 - 3x + 5} \\
 \underline{-x^3 + x^2 - x} \\
 2x^2 - 2x + 5 \\
 \underline{2x^2 - 2x + 2} \\
 3
 \end{array}$$

Fukey Education

Class : 10th mathematics
Chapter- 3: Pair of Linear Equations in Two Variables

Substitution

Solve: $7x - 15y = 2$ —(i)
 $x + 2y = 3$ —(ii)

Solution: From equation (ii), $x = 3 - 2y$
substitute value of x in eq. (i)
 $7(3 - 2y) - 15y = 2$
 $-29y = -19 \Rightarrow y = \frac{19}{29}$
Now, from $x = 3 - 2y$
 $x = 3 - 2\left(\frac{19}{29}\right) = \left(\frac{49}{29}\right)$

By Elimination

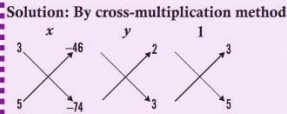
Solve: $x + 3y = 6$ —(i)
 $2x + 3y = 12$ —(ii)

Now, Adding equation (i) and (ii)
 $3x = 18$ or $x = 6$
Again, from (i) $\times 2$ —(ii)
 $3y = 0$ or, $y = 0$
Hence, $x = 6, y = 0$

By Cross Multiplication

Solve: $2x + 3y - 46 = 0$ —(i)
 $3x + 5y - 74 = 0$ —(ii)

Solution: By cross-multiplication method



Then, $\frac{x}{3(-74) - 5(-46)} = \frac{y}{(-46)(3) - (-74)(2)} = \frac{1}{2(5) - 3(3)}$

$\frac{x}{-222 + 230} = \frac{y}{-138 + 148} = \frac{1}{10 - 9}$

$\frac{x}{8} = \frac{y}{1} \Rightarrow \frac{x}{8} = \frac{y}{1}$ and $\frac{y}{10} = \frac{1}{1}$

i.e. $x = 8$ and $y = 10$

Algebraic Methods

General Form

Pair of Linear Equations in Two Variables

Solution Graphically

Graphically Presentation

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$a_1, b_1, c_1, a_2, b_2, c_2$, – Real numbers

Each solution (x, y) , corresponds to a point on the line representing the equation and vice-versa

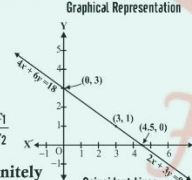
Pair of Lines = $2x + 3y - 9 = 0$
 $4x + 6y - 18 = 0$

$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{-9}{-18}$

Compare the Ratios = $\frac{a}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Algebraic Interpretation = Infinitely many solutions – Dependent

Graphical Representation



Coincident Lines

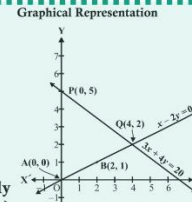
Pair of Lines = $x - 2y = 0$
 $3x + 4y - 20 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{4}, \frac{c_1}{c_2} = \frac{0}{-20}$

Compare the Ratios = $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Algebraic Interpretation : Exactly one solution – consistent (unique)

Graphical Representation



Intersecting Lines

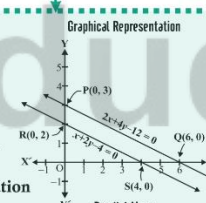
Pair of Lines = $x + 2y - 4 = 0$
 $2x + 4y - 12 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-4}{-12}$

Compare the Ratios = $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Algebraic Interpretation : No solution – Inconsistent

Graphical Representation



Parallel Lines

Important Questions

Multiple Choice questions-

1. Graphically, the pair of equations $7x - y = 5$; $21x - 3y = 10$ represents two lines which are

- (a) intersecting at one point
- (b) parallel
- (c) intersecting at two points
- (d) coincident

2. The pair of equations $3x - 5y = 7$ and $-6x + 10y = 7$ have

- (a) a unique solution
- (b) infinitely many solutions
- (c) no solution
- (d) two solutions

3. If a pair of linear equations is consistent, then the lines will be

- (a) always coincident
- (b) parallel
- (c) always intersecting
- (d) intersecting or coincident

4. The pair of equations $x = 0$ and $x = 5$ has

- (a) no solution
- (b) unique/one solution

- (c) two solutions
- (d) infinitely many solutions
5. The pair of equation $x = -4$ and $y = -5$ graphically represents lines which are
- (a) intersecting at $(-5, -4)$
- (b) intersecting at $(-4, -5)$
- (c) intersecting at $(5, 4)$
- (d) intersecting at $(4, 5)$
6. One equation of a pair of dependent linear equations is $2x + 5y = 3$. The second equation will be
- (a) $2x + 5y = 6$
- (b) $3x + 5y = 3$
- (c) $-10x - 25y + 15 = 0$
- (d) $10x + 25y = 15$
7. If $x = a$, $y = b$ is the solution of the equations $x + y = 5$ and $2x - 3y = 4$, then the values of a and b are respectively
- (a) 6, -1
- (b) 2, 3
- (c) 1, 4
- (d) $19/5$, $6/5$
8. The graph of $x = -2$ is a line parallel to the
- (a) x-axis
- (b) y-axis
- (c) both x- and y-axis
- (d) none of these
9. The graph of $y = 4x$ is a line
- (a) parallel to x-axis

- (b) parallel to y-axis
- (c) perpendicular to y-axis
- (d) passing through the origin
10. The graph of $y = 5$ is a line parallel to the
- (a) x-axis
- (b) y-axis
- (c) both axis
- (d) none of these

Very Short Questions:

- If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find value of k .
- Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions.
- Do the equations $4x + 3y - 1 = 5$ and $12x + 9y = 15$ represent a pair of coincident lines?
- Find the co-ordinate where the line $x - y = 8$ will intersect y-axis.
- Write the number of solutions of the following pair of linear equations:
 $x + 2y - 8 = 0, 2x + 4y = 16$
- Is the following pair of linear equations consistent? Justify your answer.
 $2ax + by = a, 4ax + 2by - 2a = 0; a, b \neq 0$
- For all real values of c , the pair of equations
 $x - 2y = 8, 5x + 10y = c$
have a unique solution. Justify whether it is true or false.
- Does the following pair of equations represent a pair of coincident lines? Justify your answer.

$$\frac{x}{2} + y + \frac{2}{5} = 0, 4x + 8y + \frac{5}{16} = 0.$$

9. If $x = a$, $y = b$ is the solution of the pair of equation $x - y = 2$ and $x + y = 4$, then find the value of a and b .
10. $\frac{3}{2}x + \frac{5}{3}y = 7$
 $9x - 10y = 14$

Short Questions :

1. Solve: $ax + by = a - b$ and $bx - ay = a + b$
2. Solve the following linear equations:

$$152x - 378y = -74 \text{ and } -378x + 152y = -604$$

3. Solve for x and y

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2; \quad x + y = 2ab$$

4. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

- (ii) for which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

5. Find whether the following pair of linear equations has a unique solution. If yes, find the

$$7x - 4y = 49 \text{ and } 5x - y = 57$$

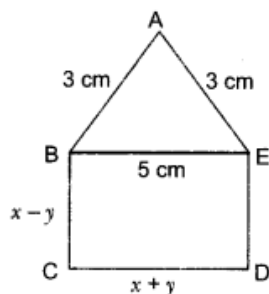
6. Solve for x and y .

$$\frac{6}{x-1} - \frac{3}{y-2} = 1; \quad \frac{5}{x-1} + \frac{1}{y-2} = 2 \text{ where } x \neq 1, y \neq 2$$

7. Solve the following pair of equations for x and y .

$$\frac{a^2}{x} - \frac{b^2}{y} = 0; \quad \frac{a^2b}{x} + \frac{b^2a}{y} = a + b, \quad x \neq 0, y \neq 0.$$

8. In $\triangle ABC$, $\angle A = x$, $\angle B = 3x$, and $\angle C = y$ if $3y - 5x = 30^\circ$, show that triangle is right angled.
9. In Fig. 3.1, ABCDE is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm. Find the value of x and y.



10. Five years ago, A was thrice as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B?

Long Questions :

- Form the pair of linear equations in this problem and find its solution graphically: 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
- Show graphically the given system of equations $2x + 4y = 10$ and $3x + 6y = 12$ has no solution.
- Solve the following pairs of linear equations by the elimination method and the substitution method:

(i) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(ii) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

- Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.
- A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay 31000 as hostel charges whereas a student B, who takes food for 26 days, pays 1180 as hostel charges. Find the fixed charges and the cost of food per day.
- Yash scored 40 marks in a test, getting 3 marks for each right answer and

losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

7. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.
8. A boat covers 25 km upstream and 44 km downstream in 9 hours. Also, it covers 15 km upstream and 22 km downstream in 5 hours. Find the speed of the boat in still water and that of the stream.

Case Study Questions:

1. A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student Anu takes food for 25 days, she has to pay ₹ 4500 as hostel charges, whereas another student Bindu who takes food for 30 days, has to pay ₹ 5200 as hostel charges.



Considering the fixed charges per month by ₹ x and the cost of food per day by ₹ y , then answer the following questions.

i. Represent algebraically the situation faced by both Anu and Bindu.

- a. $x + 25y = 4500$, $x + 30y = 5200$
- b. $25x + y = 4500$, $30x + y = 5200$
- c. $x - 25y = 4500$, $x - 30y = 5200$
- d. $25x - y = 4500$, $30x - y = 5200$

ii. The system of linear equations, represented by above situations has.

- a. No solution.
- b. Unique solution.
- c. Infinitely many solutions.
- d. None of these.

iii. The cost of food per day is:

- a. ₹ 120
- b. ₹ 130
- c. ₹ 140
- d. ₹ 1300

iv. The fixed charges per month for the hostel is:

- a. ₹ 1500
- b. ₹ 1200
- c. ₹ 1000
- d. ₹ 1300

v. If Bindu takes food for 20 days, then what amount she has to pay?

- a. ₹ 4000
- b. ₹ 3500
- c. ₹ 3600
- d. ₹ 3800

2. From Bengaluru bus stand, if Riddhima buys 2 tickets to Malleswaram and 3 tickets to Yeswanthpur, then total cost is ₹ 46; but if she buys 3 tickets to Malleswaram and 5 tickets to Yeswanthpur, then total cost is ₹ 74.



Consider the fares from Bengaluru to Malleswaram and that to Yeswanthpur as ₹ x and ₹ y respectively and answer the following questions.

i. 1st situation can be represented algebraically as:

- a. $3x - 5y = 74$
- b. $2x + 5y = 74$
- c. $2x - 3y = 46$
- d. $2x + 3y = 46$

ii. 2nd situation can be represented algebraically as:

- a. $5x + 3y = 74$
- b. $5x - 3y = 74$
- c. $3x + 5y = 74$
- d. $3x - 5y = 74$

iii. Fare from Bengaluru to Malleswaram is:

- a. ₹ 6
- b. ₹ 8
- c. ₹ 10
- d. ₹ 2

iv. Fare from Bengaluru to Yeswanthpur is:

- a. ₹ 10
- b. ₹ 12

- c. ₹ 14
 - d. ₹ 16
- v. The system of linear equations represented by both situations has:
- a. Infinitely many solutions.
 - b. No solution.
 - c. Unique solution.
 - d. None of these.

Assertion reason questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- b. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- c. (C) Assertion (A) is true but reason (R) is false.
- d. (d) Assertion (A) is false but reason (R) is true.

Assertion: The graph of the linear equations $3x+2y=12$ and $5x-2y=4$ gives a pair of intersecting lines.

Reason: The graph of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ gives a pair of intersecting lines if $a_1/a_2 \neq b_1/b_2$

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- c. Assertion (A) is true but reason (R) is false.
- d. Assertion (A) is false but reason (R) is true.

Assertion: If the pair of lines are coincident, then we say that pair of lines is consistent and it has a unique solution.

Reason: If the pair of lines are parallel, then the pairs has no solution and is called inconsistent pair of equations.

Answer Key-

Multiple Choice questions-

1. (b) -10
2. (d) $a = 0$, $b = -6$
3. (d) more than 3
4. (a) $b - a + 1$
5. (b) both negative
6. (a) cannot both be positive
7. (c) c and a have the same sign
8. (a) has no linear term and the constant term is negative.
9. (d) more than 4
10. (b) $x^2 + 9x + 20$
11. (a) both negative

Very Short Answer :

1. Since the given lines are parallel

$$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1} \quad \text{i.e., } k = \frac{15}{4}$$

2. The given system of equations will have infinitely many solutions if $\frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$ which is not possible

\therefore For no value of c , the given system of equations have infinitely many solutions.

- 3.

$$\text{Here, } \frac{4}{12} = \frac{3}{9} \neq \frac{6}{15} \quad \text{i.e., } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Given equations do not represent a pair of coincident lines.

4. The given line will intersect y-axis when $x = 0$.

$$\therefore 0 - y = 8 \Rightarrow y = -8$$

Required coordinate is $(0, -8)$.

- 5.

$$\text{Here, } \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore The given pair of linear equations has infinitely many solutions.

6. Yes,

$$\text{Here, } \frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore The given system of equations is consistent.

- 7.

$$\text{Here, } \frac{a_1}{a_2} = \frac{1}{5}, \quad \frac{b_1}{b_2} = \frac{-2}{+10} = \frac{-1}{5}, \quad \frac{c_1}{c_2} = \frac{8}{c}$$

$$\text{Since } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

- 8.

$$\text{Here, } a_1 = \frac{1}{2}, \quad b_1 = 1, \quad c_1 = \frac{2}{5} \quad \text{and} \quad a_2 = 4, \quad b_2 = 8, \quad c_2 = \frac{5}{16}$$

$$\frac{a_1}{a_2} = \frac{\frac{1}{2}}{4} = \frac{1}{8}, \quad \frac{b_1}{b_2} = \frac{1}{8}, \quad \frac{c_1}{c_2} = \frac{\frac{2}{5}}{\frac{5}{16}} = \frac{32}{25}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

\therefore The given system does not represent a pair of coincident lines.

$$x - y = 2 \dots (i)$$

$$x + y = 4 \dots (ii)$$

9. On adding (i) and (ii), we get $2x = 6$ or $x = 3$

$$\text{From (i), } 3 - y \Rightarrow 2 = y = 1$$

$$a = 3, b = 1.$$

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$, and $\frac{c_1}{c_2}$ find out whether the following pair of linear equations consistent or inconsistent. is consistent or inconsistent.

10.

$$\text{We have, } \frac{3}{2}x + \frac{5}{3}y = 7 \dots (i)$$

$$9x - 10y = 14 \dots (ii)$$

$$\text{Here, } a_1 = \frac{3}{2}, \quad b_1 = \frac{5}{3}, \quad c_1 = 7$$

$$a_2 = 9, \quad b_2 = -10, \quad c_2 = 14$$

$$\text{Thus, } \frac{a_1}{a_2} = \frac{3}{2 \times 9} = \frac{1}{6}, \quad \frac{b_1}{b_2} = \frac{5}{3 \times (-10)} = -\frac{1}{6}$$

$$\text{Hence, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}. \text{ So, it has unique solution and it is consistent.}$$

Short Answer :

1. The given system of equations may be written as

$$ax + by - (a - b) = 0$$

$$bx - ay - (a + b) = 0$$

By cross-multiplication, we have

$$\frac{x}{b \times -(a+b) - (-a) \times -(a-b)} = \frac{-y}{a \times -(a+b) - b \times -(a+b)} = \frac{1}{a \times b - b \times -a}$$

$$\Rightarrow \frac{x}{b \times -(a+b) - (-a) \times -(a-b)} = \frac{-y}{a \times -(a+b) - b \times -(a+b)} = \frac{1}{-a^2 - b^2}$$

$$\Rightarrow \frac{x}{-b(a+b) - a(a-b)} = \frac{-y}{-a(a+b) + b(a-b)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 - b^2} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \frac{x}{-(a^2 + b^2)} = \frac{y}{(a^2 + b^2)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow x = -\frac{(a^2 + b^2)}{-(a^2 + b^2)} = 1 \quad \text{and} \quad y = \frac{(a^2 + b^2)}{-(a^2 + b^2)} = -1$$

(21)

Hence, the solution of the given system of equations is $x = 1, y = -1$

2. We have, $152x - 378y = -74 \dots(i)$

$$-378x + 152y = -604 \dots(ii)$$

Adding equation (i) and (ii), we get

$$\begin{array}{r} 152x - 378y = -74 \\ -378x + 152y = -604 \\ \hline -226x - 226y = -678 \end{array}$$

$$\Rightarrow -226(x + y) = -678$$

$$\Rightarrow x + y = \frac{-678}{-226}$$

$$\Rightarrow x + y = 3 \dots(iii)$$

Subtracting equation (ii) from (i), we get

$$\begin{array}{r} 152x - 378y = -74 \\ -378x + 152y = -604 \\ + \quad - \quad + \\ \hline 530x - 530y = 530 \end{array}$$

$$\Rightarrow x - y = 1 \dots(iv)$$

Adding equations (iii) and (iv), we get

$$\begin{array}{r} x + y = 3 \\ x - y = 1 \\ \hline 2x = 4 \end{array} \Rightarrow x = 2$$

Putting the value of x in (iii), we get

$$2 + y = 3 \Rightarrow y = 1$$

Hence, the solution of given system of equations is $x = 2, y = 1$.

3.

We have, $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2 \dots(i)$

$$x + y = 2ab \dots(ii)$$

Multiplying (ii) by b/a , we get

$$\frac{b}{a}x + \frac{b}{a}y = 2b^2 \dots(iii)$$

Subtracting (iii) from (i), we get

$$\left(\frac{a}{b} - \frac{b}{a}\right)y = a^2 + b^2 - 2b^2 \Rightarrow \left(\frac{a^2 - b^2}{ab}\right)y = (a^2 - b^2)$$

$$\Rightarrow y = (a^2 - b^2) \times \frac{ab}{(a^2 - b^2)} \Rightarrow y = ab$$

4. (i) We have, $2x + 3y = 7$

$$(a - b)x + (a + b)y = 3a + b - 2 \dots \text{(ii)}$$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = 7$ and

$$a_2 = a - b, b_2 = a + b, c_2 = 3a + b - 2$$

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Now, $\frac{2}{a-b} = \frac{3}{a+b}$

$$\Rightarrow 2a + 2b = 3a - 3b \Rightarrow 2a - 3a = -3b - 2b$$

$$\Rightarrow -a = -5b \dots \text{(iii)}$$

$$\therefore a = 5b$$

Again, we have

$$\frac{3}{a+b} = \frac{7}{3a+b-2} \Rightarrow 9a + 3b - 6 = 7a + 7b$$

$$\Rightarrow 9a - 7a + 3b - 7b - 6 = 0 \Rightarrow 2a - 4b - 6 = 0 \Rightarrow 2a - 4b = 6$$

$$\Rightarrow a - 2b = 3 \dots \text{(iv)}$$

Putting $a = 5b$ in equation (iv), we get

$$5b - 2b = 3 \text{ or } 3b = 3 \text{ i.e., } b = \frac{3}{3} = 1$$

Putting the value of b in equation (ii), we get $a = 5(1) = 5$

Hence, the given system of equations will have an infinite number of solutions for $a = 5$ and $b = 1$.

(ii) We have, $3x + y = 1$, $3x + y - 1 = 0 \dots \text{(i)}$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

$$\Rightarrow (2k - 1)x + (k - 1)y - (2k + 1) = 0 \dots\dots(ii)$$

Here, $a_1 = 3, b_1 = 1, C_1 = -1$

$a_2 = 2k - 1, b_2 = k - 1, C_2 = -(2k + 1)$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{3}{2k - 1} = \frac{1}{k - 1} \neq \frac{1}{2k + 1}$$

Now, $\frac{3}{2k - 1} = \frac{1}{k - 1} \Rightarrow 3k - 3 = 2k - 1$

$$\Rightarrow 3k - 2k = 3 - 1 \Rightarrow k = 2$$

5. Hence, the given system of equations will have no solutions for $k = 2$.

We have, $7x - 4y = 49 \dots\dots(i)$

and $5x - 6y = 57 \dots\dots(ii)$

Here, $a_1 = 7, b_1 = -4, c_1 = 49$

$a_2 = 5, b_2 = -6, c_2 = 57$

So, $\frac{a_1}{a_2} = \frac{7}{5}, \frac{b_1}{b_2} = \frac{-4}{-6} = \frac{2}{3}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, system has a unique solution.

Multiply equation (i) by 5 and equation (ii) by 7 and subtract

$$\begin{array}{r} 35x - 20y = 245 \\ -35x + 42y = -399 \\ \hline 22y = -154 \end{array} \Rightarrow y = -7$$

Put $y = -7$ in equation (ii)

$$5x - 6(-7) = 57 \Rightarrow 5x = 57 - 42 \Rightarrow x = 3$$

hence, $x = 3$ and $y = -7$.

- 6.

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

The given equations become

$$6p - 3q = 1 \quad \dots(i)$$

$$5p + q = 2 \quad \dots(ii)$$

Multiply equation (ii) by 3 and add in equation (i)

$$15p + 3q = 6$$

$$\frac{6p - 3q = 1}{21p = 7}$$

$$\Rightarrow p = \frac{7}{21} = \frac{1}{3}$$

Putting this value in equation (i) we get

$$6 \times \frac{1}{3} - 3q = 1 \quad \Rightarrow \quad 2 - 3q = 1 \quad \Rightarrow \quad 3q = 1, \quad \Rightarrow \quad q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3} \quad \Rightarrow \quad x-1 = 3 \quad \Rightarrow \quad x = 4$

$$\frac{1}{y-2} = q = \frac{1}{3} \quad \Rightarrow \quad y-2 = 3 \quad \Rightarrow \quad y = 5$$

Hence, $x = 4$ and $y = 5$.

7.

$$\frac{a^2b}{x} + \frac{b^2a}{y} = a + b \quad \dots(ii)$$

Multiply equation (i) by a and adding to equation (ii)

$$\frac{a^2a}{x} - \frac{b^2a}{y} + \frac{a^2b}{x} + \frac{b^2a}{y} = 0 + (a+b)$$

$$\Rightarrow \frac{a^3}{x} + \frac{a^2b}{x} = a + b \quad \Rightarrow \quad \frac{a^2}{x}(a+b) = a+b \Rightarrow x = \frac{a^2(a+b)}{a+b} = a^2$$

Putting the value of x in equation (i), we get

$$\frac{a^2}{a^2} - \frac{b^2}{y} = 0 \quad \Rightarrow \quad 1 - \frac{b^2}{y} = 0 \quad \Rightarrow \quad \frac{b^2}{y} = 1 \quad \Rightarrow \quad y = b^2$$

Hence, $x = a^2, y = b^2$.

8. $\angle A + 2B + \angle C = 180^\circ$

(Sum of interior angles of A ABC) $x + 3x + y = 180^\circ$

$$4x + y = 180^\circ \dots(i)$$

$3y - 5x = 30^\circ$ (Given) ... (ii) Multiply equation (i) by 3 and subtracting from eq. (ii), we get

$$-17x = -510 \Rightarrow x = 910 = 30^\circ$$

$$17 \text{ then } \angle A = x = 30^\circ \text{ and } 2B = 3x = 3 \times 30^\circ = 90^\circ$$

$$\angle C = y = 180^\circ - (\angle A + \angle B) = 180^\circ - 120^\circ = 60^\circ$$

$\angle A = 30^\circ$, $\angle B = 90^\circ$, $\angle C = 60^\circ$ Hence ΔABC is right triangle right angled at B.

9. Since $BC \parallel DE$ and $BE \parallel CD$ with $BC \parallel CD$.

BCDE is a rectangle.

Opposite sides are equal $BE = CD$

$$\therefore x + y = 5 \dots\dots (i)$$

$$DE = BC = x - y$$

Since perimeter of ABCDE is 21 cm.

$$AB + BC + CD + DE + EA = 21$$

$$3 + x - y + x + y + x - y + 3 = 21 \Rightarrow 6 + 3x - y = 21$$

$$3x - y = 15 \dots\dots (iii)$$

Adding (i) and (ii), we get

$$4x = 20 \Rightarrow x = 5$$

On putting the value of x in (i), we get $y = 0$

Hence, $x = 5$ and $y = 0$.

10. Let the present ages of B and A be x years and y years respectively. Then

B's age 5 years ago = $(x - 5)$ years

and A's age 5 years ago = $(y - 5)$ years

$$(y - 5) = 3(x - 5) = 3x - y = 10 \dots\dots (i)$$

B's age 10 years hence = $(x + 10)$ years

A's age 10 years hence = $(y + 10)$ years

$$y + 10 = 2(x + 10) = 2x - y = -10 \dots\dots (ii)$$

On subtracting (ii) from (i) we get $x = 20$

Putting $x = 20$ in (i) we get

$$(3 \times 20) - y = 10 \Rightarrow y = 50$$

$$\therefore x = 20 \text{ and } y = 50$$

Hence, B's present age = 20 years and A's present age = 50 years.

Long Answer :

1. Let x be the number of girls and y be the number of boys.

According to question, we have

$$x = y + 4$$

$$\Rightarrow x - y = 4 \dots\dots(i)$$

Again, total number of students = 10

$$\text{Therefore, } x + y = 10 \dots(ii)$$

Hence, we have following system of equations

$$x - y = 4$$

$$\text{and } x + y = 10$$

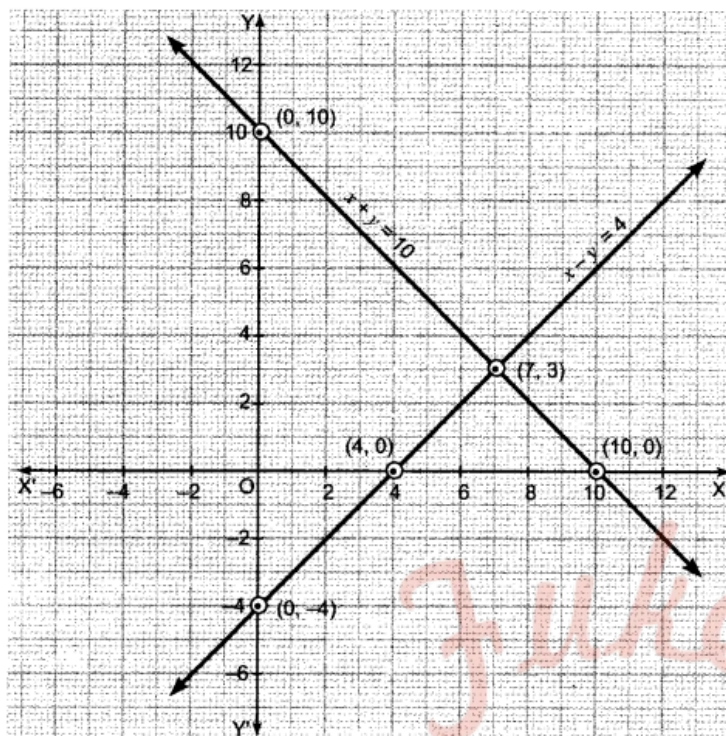
From equation (i), we have the following table:

x	0	4	7
y	-4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have



Here, the two lines intersect at point $(7, 3)$ i.e., $x = 7$, $y = 3$.

So, the number of girls = 7

and number of boys = 3.

2. We have, $2x + 4y = 10$

$$\Rightarrow 4y = 10 - 2x \Rightarrow y = \frac{5-x}{2}$$

Thus, we have the following table:

x	1	3	5
y	2	1	0

Plot the points A (1, 2), B (3, 1) and C (5, 0) on the graph paper. Join A, B and C and extend it on both sides to obtain the graph of the equation $2x + 4y = 10$.

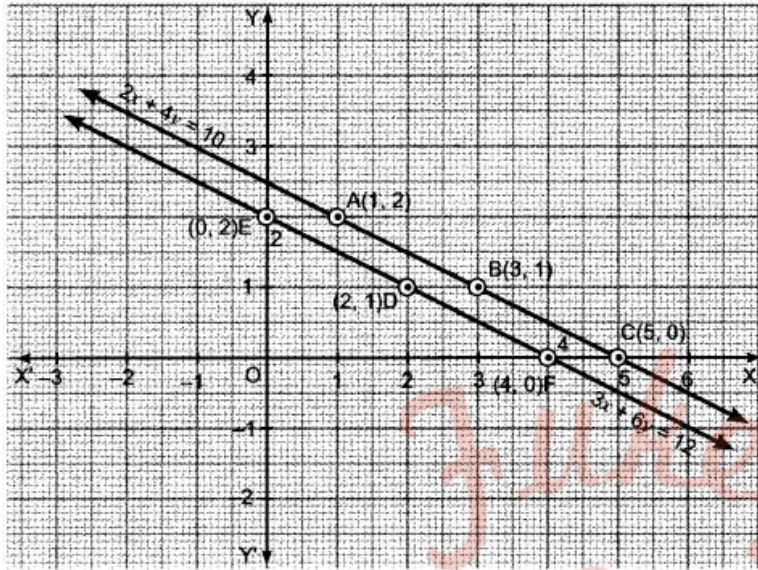
We have, $3x + 6y = 12$

$$\Rightarrow 6y = 12 - 3x \Rightarrow y = \frac{4-x}{2}$$

Thus, we have the following table :

x	2	0	4
y	1	2	0

Plot the points D (2, 1), E (0, 2) and F (4,0) on the same graph paper. Join D, E and F and extend it on both sides to obtain the graph of the equation $3x + 6y = 12$.



We find that the lines represented by equations $2x + 4y = 10$ and $3x + y = 12$ are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

3. (i) We have, $3x - 5y - 4 = 0$

$$\Rightarrow 3x - 5y = 4 \dots\dots(i)$$

Again, $9x = 2y + 7$

$$9x - 2y = 7 \dots(ii)$$

By Elimination Method:

Multiplying equation (i) by 3, we get

$$9x - 15y = 12 \dots (iii)$$

Subtracting (ii) from (iii), we get

$$\begin{array}{r} 9x - 15y = 12 \\ 9x - 2y = 7 \\ \hline -13y = 5 \end{array}$$

$$\Rightarrow y = -\frac{5}{13}$$

Putting the value of y in equation (ii), we have

$$9x - 2\left(-\frac{5}{13}\right) = 7 \Rightarrow 9x + \frac{10}{13} = 7 \Rightarrow 9x = 7 - \frac{10}{13}$$

$$\Rightarrow 9x = \frac{91 - 10}{13} \Rightarrow 9x = \frac{81}{13} \Rightarrow x = \frac{9}{13}$$

Hence, the required solution is $x = \frac{9}{13}$, $y = -\frac{5}{13}$

By Substitution Method:

Expressing x in terms of y from equation (i), we have

$$x = \frac{4 + 5y}{3}$$

Substituting the value of x in equation (ii), we have

$$9 \times \left(\frac{4 + 5y}{3}\right) - 2y = 7$$

$$\Rightarrow 3 \times (4 + 5y) - 2y = 7$$

$$\Rightarrow 12 + 15y - 2y = 7 \Rightarrow 13y = 7 - 12$$

$$\therefore y = -\frac{5}{13}$$

Putting the value of y in equation (i), we have

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4 \Rightarrow 3x + \frac{25}{13} = 4$$

$$\Rightarrow 3x = 4 - \frac{25}{13} \Rightarrow 3x = \frac{27}{13}$$

$$\therefore x = \frac{9}{13}$$

Hence, the required solution is $x = \frac{9}{13}$, $y = -\frac{5}{13}$.

$$(ii) \text{ We have, } \frac{x}{2} + \frac{2y}{3} = -1 \Rightarrow \frac{3x + 4y}{6} = -1$$

$$\therefore 3x + 4y = -6 \quad \dots(i)$$

$$\text{and } x - \frac{y}{3} = 3 \Rightarrow \frac{3x - y}{3} = 3$$

$$\therefore 3x - y = 9 \quad \dots(ii)$$

By Elimination Method:

Subtracting (ii) from (i), we have

$$5y = -15 \text{ or } y = -3$$

Putting the value of y in equation (i), we have

$$3x + 4 \times (-3) = -6 \Rightarrow 3x = -6 + 12$$

$$\therefore 3x - 12 = -6 \Rightarrow 3x = 6$$

$$\therefore x = 6 \div 3 = 2$$

Hence, solution is $x = 2, y = -3$.

By Substitution Method:

Expressing x in terms of y from equation (i), we have

$$3 \times \left(\frac{-6-4y}{3} \right) - y = 9 \Rightarrow -6 - 4y - y = 9 \Rightarrow -6 - 5y = 9$$

Substituting the value of x in equation (ii), we have

$$\therefore -5y = 9 + 6 = 15$$

$$y = -\frac{15}{5} = -3$$

Putting the value of y in equation (i), we have

$$3x + 4 \times (-3) = -6 \Rightarrow 3x - 12 = -6$$

$$\therefore 3x = 12 - 6 = 6$$

$$\therefore x = \frac{6}{3} = 2$$

Hence, the required solution is $x = 2, y = -3$.

4. We have, $x - y + 1 = 0$ and $3x + 2y - 12 = 0$

Thus, $x - y = -1 \Rightarrow x = y - 1 \dots(i)$

$$3x + 2y = 12 \Rightarrow x = \frac{12-2y}{3} \dots (ii)$$

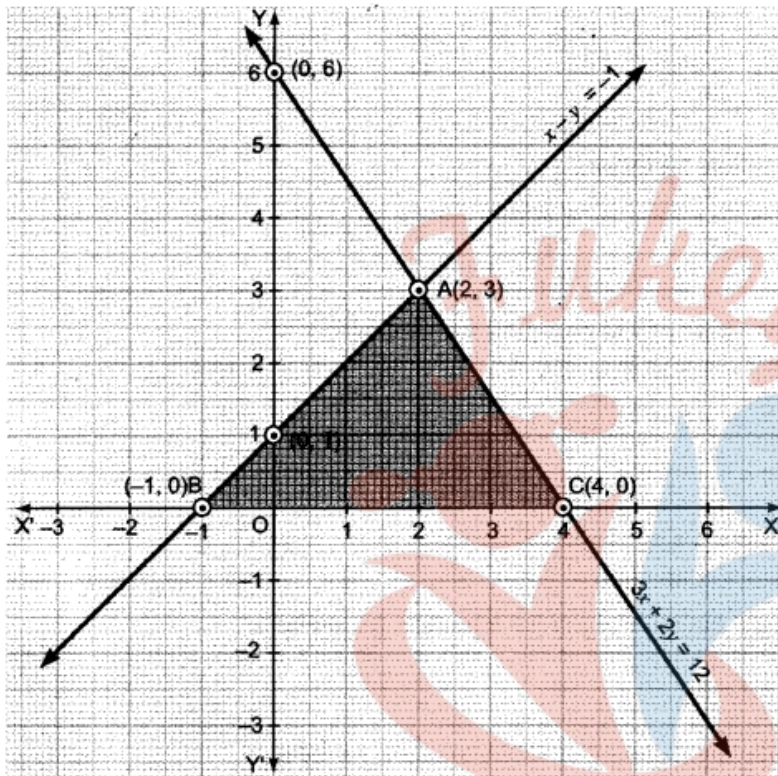
From equation (i), we have

x	-1	0	2
y	0	1	3

From equation (ii), we have

x	0	4	2
y	6	0	3

Plotting this, we have



ABC is the required (shaded) region and point of intersection is (2, 3).

∴ The vertices of the triangle are (-1, 0), (4, 0), (2, 3).

5. Let the fixed charge be *x and the cost of food per day be by.

Therefore, according to question,

$$x + 20y = 1000 \dots(i)$$

$$x + 26y = 1180 \dots(ii)$$

Now, subtracting equation (ii) from (i), we have

$$\begin{array}{r}
 x + 20y = 1000 \\
 \underline{x + 26y = 1180} \\
 -6y = -180 \\
 \therefore y = \frac{-180}{-6} = 30
 \end{array}$$

Putting the value of y in equation (i), we have

$$x + 20 \times 30 = 1000 \Rightarrow x + 600 = 1000 \Rightarrow x = 1000 - 600 = 400$$

Hence, fixed charge is ₹400 and cost of food per day is ₹30.

6. Let x be the number of questions of right answer and y be the number of questions of wrong answer.

According to question,

$$3x - y = 40 \dots (i)$$

$$\text{and } 4x - 2y = 50$$

$$\text{or } 2x - y = 25 \dots (ii)$$

Subtracting (ii) from (i), we have

$$\begin{array}{r} 3x - y = 40 \\ - 2x - y = 25 \\ \hline x = 15 \end{array}$$

Putting the value of x in equation (i), we have

$$3 \times 15 - y = 40 \Rightarrow 45 - y = 40$$

$$\therefore y = 45 - 40 = 5$$

Hence, total number of questions is $x + y$, i.e., $5 + 15 = 20$.

7. Let one man alone can finish the work in x days and one boy alone can finish the work in y days

$$\text{Then, One day work of one man} = \frac{1}{x}, \text{ One day work of one boy} = \frac{1}{y}$$

$$\therefore \text{One day work of 8 men} = \frac{8}{x}, \text{ One day work of 12 boys} = \frac{12}{y}$$

Since 8 men and 12 boys can finish the work in 10 days

$$10\left(\frac{8}{x} + \frac{12}{y}\right) = 1 \Rightarrow \frac{80x}{x} + \frac{120}{y} = 1 \quad \dots(i)$$

Again, 6 men and 8 boys can finish the work in 14 days

$$\therefore 14\left(\frac{6}{x} + \frac{8}{y}\right) = 1 \Rightarrow \frac{84}{x} + \frac{112}{y} = 1 \quad \dots(ii)$$

Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in equations (i) and (ii), we get

$$80u + 120v - 1 = 0 \quad \text{and} \quad 84u + 112v - 1 = 0$$

By using cross-multiplication, we have

$$\frac{u}{120 \times -1 - 112 \times -1} = \frac{-v}{80 \times -1 - 84 \times -1} = \frac{1}{80 \times 112 - 84 \times 120}$$

$$\Rightarrow \frac{u}{-120 + 112} = \frac{-v}{-80 + 84} = \frac{1}{8960 - 10080}$$

$$\Rightarrow \frac{u}{-8} = \frac{-v}{4} = \frac{1}{-1120}$$

Hence, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

8. Let the speed of the boat in still water be x km/h and that of the stream be y km/h. Then,

Speed upstream $(x - y)$ km/h

Speed downstream $(x + y)$ km/h

Now, time taken to cover 25 km upstream = $\frac{25}{x - y}$ hours

Time taken to cover 44 km downstream = $\frac{44}{x + y}$ hours

The total time of journey is 9 hours

$$\frac{25}{x - y} + \frac{44}{x + y} = 9 \quad \dots(i)$$

Time taken to cover 15 km upstream = $\frac{15}{x - y}$

Time taken to cover 22 km downstream = $\frac{22}{x + y}$

In this case, total time of journey is 5 hours.

$$\therefore \frac{15}{x-y} + \frac{22}{x+y} = 5 \quad \dots(ii)$$

Put $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (i) and (ii), we get

$$25u + 44v = 9 \Rightarrow 25u + 44v - 9 = 0 \dots(iii)$$

$$15u + 22v = 5 \Rightarrow 15u + 22v - 5 = 0 \dots(iv)$$

By cross-multiplication, we have

$$\Rightarrow u = \frac{22}{110} = \frac{1}{5} \quad \text{and} \quad v = \frac{1}{11}$$

$$\text{We have, } u = \frac{1}{5} \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x-y = 5 \quad \dots(v)$$

$$\text{and } v = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x+y = 11 \quad \dots(vi)$$

$$\Rightarrow u = \frac{22}{110} = \frac{1}{5} \quad \text{and} \quad v = \frac{1}{11}$$

$$\text{We have, } u = \frac{1}{5} \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x-y = 5 \quad \dots(v)$$

$$\text{and } v = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x+y = 11 \quad \dots(vi)$$

Solving equations (v) and (vi), we get $x = 8$ and $y = 3$.

Hence, speed of the boat in still water is 8 km/h and speed of the stream is 3 km/h.

Case Study Answers:

1. Answer :

i. (a) $x + 25y = 4500$, $x + 30y = 5200$

Solution:

For student Anu:

Fixed charge + cost of food for 25 days = ₹ 4500

i.e., $x + 25y = 4500$

For student Bindu:

Fixed charges + cost of food for 30 days = ₹ 5200

i.e., $x + 30y = 5200$

ii. (b) Unique solution.

Solution:

From above, we have $a_1 = 1$, $b_1 = 25$

$c_1 = -4500$ and $a_2 = 1$, $b_2 = 30$, $c_2 = -5200$

$$\therefore \frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = \frac{25}{30} = \frac{5}{6}, \frac{c_1}{c_2} = \frac{-4500}{-5200} = \frac{45}{52}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, system of linear equations has unique solution.

iii. (c) ₹ 140

Solution:

We have, $x + 25y = 4500$

and $x + 30y = 5200$

Subtracting (i) from (ii), we get

$$5y = 700 \Rightarrow y = 140$$

\therefore Cost of food per day is ₹ 140

iv. (c) ₹ 1000

Solution:

We have, $x + 25y = 4500$

$$\Rightarrow x = 4500 - 25 \times 140$$

$$\Rightarrow x = 4500 - 3500 = 1000$$

∴ Fixed charges per month for the hostel is ₹ 100

v. (d) ₹ 3800

Solution:

We have, $x = 1000$, $y = 140$ and Bindu takes food for 20 days.

∴ Amount that Bindu has to pay = ₹ $(1000 + 20 \times 140) = ₹ 3800$

2. Answer :

i. (d) $2x + 3y = 46$

Solution:

1st situation can be represented algebraically as.

$$2x + 3y = 46$$

ii. (c) $3x + 5y = 74$

Solution:

2nd situation can be represented algebraically as:

$$3x + 5y = 74$$

iii. (b) ₹ 8

Solution:

We have, $2x + 3y = 46$(i)

$3x + 5y = 74$(ii)

Multiplying (i) by 5 and (ii) by 3 and then subtracting, we get

$$10x - 9x = 230 - 222 \Rightarrow x = 8$$

∴ Fare from Bengaluru to Malleswaram is ₹ 8.

iv. (a) ₹ 10

Solution:

Putting the value of x in equation (i), we get

$$3y = 46 - 2 \times 8 = 30 \Rightarrow y = 10$$

\therefore Fare from Bengaluru to Yeswanthpur is ₹ 10

v. (c) Unique solution.

Solution:

We have, $a_1 = 2$, $b_1 = 3$, $c_1 = -46$ and

$$a_2 = 3, b_2 = 5, C_2 = -74$$

$$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{3}{5}, \frac{c_1}{c_2} = \frac{-46}{-74} = \frac{23}{37}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus system of linear equations has unique solution.

Assertion reason Answer-

1. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
2. (d) Assertion (A) is true but reason (R) is false.

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