MATHEMATICS

Chapter 3: Matrices

MATRICES

Top Terms

1. A matrix is an ordered rectangular array of numbers (real or complex) or functions or names or any type of data. The numbers or functions are called the elements or the entries of the matrix.

2. The horizontal lines of elements constitute the rows of the matrix and the vertical lines of elements constitute the columns of the matrix.

3. Each number or entity in a matrix is called its element.

4. If a matrix contains m rows and n columns, then it is said to be a matrix of the order m × n (read as m by n).

5. The total number of elements in a matrix is equal to the product of its number of rows and number of columns.

6. A matrix is said to be a column matrix if it has only one column.

7. $A = [a_{ij}]_{m \times 1}$ matrix is said to be a row matrix if it has only one row.

8. A matrix is said to be a row matrix if it has only one row.

9. $B = [b_{ij}]_{1 \times n}$ is row matrix of order 1 × n.

10. **Rectangular matrix:** A matrix in which the number of rows is not equal to the number of columns is called a rectangular matrix.

11. A matrix each of whose elements is zero is called a zero matrix or null matrix.

12. A matrix in which the number of rows is equal to the number of columns is said to be a square matrix. A matrix of order 'm × n' is said to be a square matrix if m = n and is known as a square matrix of order 'n'.

13. A square matrix which has every non—diagonal element as zero is called a diagonal matrix.

14. A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a diagonal matrix if all its non-diagonal elements are zero, i.e., a matrix $A = [a_{ij}]_{m \times m}$ is said to be a diagonal matrix if $a_{ij} = 0$ when $i \neq j$.

15. A square matrix in which the elements in the diagonal are all 1 and the rest are all zero is called an identity matrix. A square matrix A = $[a_{ij}]_{n \times n}$ is an matrix if $a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

16. A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is a square matrix B = $[b_{ij}]_{n \times n}$ is said to be a scaler matrix if b_{ij} = 0 when i ≠ j and b_{ij} = k when i = j for some constant k.

17. **Upper triangular matrix:** A square matrix A = $[a_{ij}]$ is called an upper triangular matrix if a_{ij} = 0 for all i > j. In an upper triangular matrix, all elements below the main diagonal are zero.

18. **Lower triangular matrix:** A square matrix A = [a„] is called a lower triangular matrix if a_{ij} = 0 for all i < j. In a lower triangular matrix, all elements above the main diagonal are zero.

19. Two matrices are said to be equal if they are of the same order and have the same corresponding elements.

20. Two matrices A = $[a_{ij}]$ and B = $[b_{ij}]$ are said to be equal if they are of the same order. Each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

21. If A is a matrix, then its transpose is obtained by interchanging its rows and columns. Transpose of a matrix A is denoted by A^t. If A = $[a_{ij}]$ be an n × m matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A' or (A^T). Theft is, $(A^T)_{ij} = a_{ji}$ for all l = 1, 2, ... m; j = 1, 2, ... n.

22. If A = $[a_{ij}]_{nxn}$ is an n × n matrix such that A^T = A, then A is called a symmetric matrix. In a symmetric matrix, $a_{ij} = a_{ji}$ for all i and j.

23. If A = $[a_{ij}]_{nxn}$ is an n × n matrix such that A^T = -A, then A is called a skew-symmetric matrix. In a skew-symmetric matrix, $a_{ij} = a_{ji}$.

24. All main diagonal elements of a skew-symmetric matrix are zero.

25. Every square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.

26. All positive integral powers of a symmetric matrix are symmetric.

27. All odd positive integral powers of a skew-symmetric matrix are skew-symmetric.

28. Let A and B be two square matrices of the order n such that AB = BA = I.

Then A is called the inverse of B and is denoted by B = A^{-1}. If B is the inverse of A, then A is also the inverse of B.

29. If A and B are two invertible matrices of the same order, then $(AB)^{-1} = B^{-1} A^{-1}$.

Top Concepts

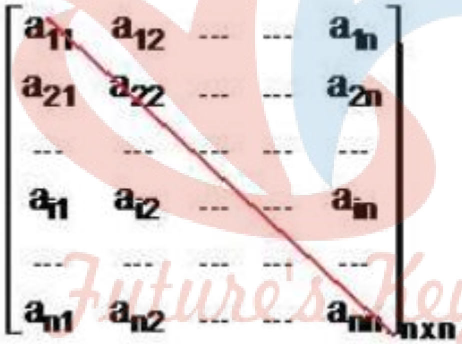
1. The order of a matrix gives the number of rows and columns present in the matrix.

2. If a matrix A has m rows and n columns, then it is denoted by $A = [a_{ij}]m \times n$. Here a_{ij} is i-j th or (i, j)th element of the matrix.

3. The simplest classification of matrices is based on the order of the matrix.

4. In case of a square matrix, the collection of elements a_{11}. a_{22} and so on constitute the Principal Diagonal or simply the diagonal of the matrix.

5. The diagonal is defined only in the case of square matrices.



6. Two matrices of the same order are comparable matrices.

7. If $A = [a_{ij}]_{m \times n}$ and B = $= [b_{ij}]_{m \times n}$ are two matrices of the order m × n, then their sum is defined as a matrix $C = [c_{ij}]_{m \times n}$ where $c_{ij} = a_{ij} + b_{ij}$ for $1 \leq i \leq m, 1 \leq j \leq n$.

8. Two matrices can be added (or subtracted) if they are of the same order.

9. For multiplying two matrices A and B, the number of columns in A must be equal to the number of rows in B.

10. If $A = [a_{ij}]_{m \times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.

Hence, $kA = [ka_{ij}]_{m \times n}$

11. If $A = [a_{ij}]_{m \times n}$ and B = $= [b_{ij}]_{m \times n}$ are are two matrices, then their difference is represented as A – B = A + (-1)B.

12. Properties of matrix addition

- Matrix addition is commutative, i.e., A + B = B + A
- Matrix addition is associative, i.e. (A + B) + C = A + (B + C)
- Existence of additive identity: Null matrix is the identity with respect to addition of matrices.

  Given a matrix $A = [a_{ij}]_{m \times n}$, there will be a corresponding null matrix O of the same order such that A + O = O + A = A
- The existence of additive inverse: Let $A = [a_{ij}]_{m \times n}$ be any matrix, then there exists another matrix $-A = -[a_{ij}]_{m \times n}$ Such that

  A + (-A) = (-A) + A = O.

13. **Cancellation law:** If A, B and C are three matrices of the same order, then

$$A + B = A + C \Rightarrow B = C \text{ and}$$
$$B + A = C + A \Rightarrow B = C$$

14. Properties of scalar multiplication of matrices

If $A = [a_{ij}]$, $B = [b_{ij}]$ are two matrices, and k and L are real number, then

i. k(A + B) = kA + kB

ii. (k + l)A = kA + lA

iii. $k(A + B) = k([a_{ij}]+[b_{ij}]) = k[a_{ij}] + k[b_{ij}] = kA + kB$

iv. $(k + L)A = (k + L)[a_{ij}] = [(k + L)a_{ij}] = k[a_{ij}] + L[a_{ij}] = kA + LA$

15. If $A = [a_{ij}]_{mxp}$, $B = [b_{ij}]_{pxn}$ are two matrices, then their product AB is given by $C = [c_{ij}]_{mxn}$ such that

$$c_{ij} = \sum_{k=1}^{p} a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{ip}b_{pj}.$$

In order to multiply two matrices A and B, the number of columns in A = number of rows in B.

16. Properties of Matrix Multiplication

Commutative law does not hold in matrices, whereas associative and distributive laws hold for matrix multiplication.

i. In general, AB ≠ BA

ii. Matrix multiplication is associative A(BC) = (AB)C

iii. Distributive laws:

A(B + C) = AB + AC

(A + B)C = AC + BC

17. The multiplication of two non-zero matrices can result in a null matrix.

18. If A is a square matrix, then we define A1 = A and $A^{n+1} = A^n . A$

19. If A is a square matrix, $a_0, a_1, a_2, ..., a_n$ are constants, then $a_0A^n + a_1A^{n-1} + a_2A^{n-2} + ... + a_{n-1}A + a_n$ is called a matrix polynomial.

20. If A, B and C are matrices, then AB = AC, A ≠ 0 ⇒ B = C.

In general, the cancellation law is not applicable in matrix multiplication.

21. Properties of transpose of matrices

i. If A is a matrix, then (AT)T = A

ii. $(A + B)^T = A^T + B^T$

iii. $(kB)^T = kB^T$, where k is any constant.

22. If A and B are two matrices such that AB exists, then $(AB)^T = B^T A^T$.

23. If A, B and C are two matrices such that AB exists, then $(ABC)^T = C^T B^T A^T$.

24. Every square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix, i.e. $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ for any square matrix A.

25. A square matrix A is called an orthogonal matrix when $AA^T = A^TA = I$.

26. A null matrix is both symmetric and skew symmetric.

27. Multiplication of diagonal matrices of the same order will be commutative.

28. There are six elementary operations on matrices—three on rows and three on columns. The first operation is interchanging the two rows, i.e., $R_i \leftrightarrow R_j$ implies that the ith row is interchanged with the jth row. The two rows are interchanged with one another and the rest of the matrix remains the same.

29. The second operation on matrices is to multiply a row with a scalar or a real number, i.e., $R_i \leftrightarrow kR_i$ that ith row of a matrix A is multiplied by k.

30. The third operation is the addition to the elements of any row, the corresponding elements of any other row multiplied by any non-zero number, i.e., $R_i \rightarrow R_i + kR_i$ k multiples of the jth row elements are added to the ith row elements.

31. Column operation on matrices ar,

i. Interchanging the two columns: $C_r \leftrightarrow C_k$ indicates that the rth column is interchanged with the kth column.

ii. Multiply a column with a non-zero constant, i.e., $C_i \rightarrow kC_i$

iii. Addition of a scalar multiple of any column to another column, i.e. $C_i \rightarrow C_j + kC_j$

32. Elementary operations help in transforming a square matrix to an identity matrix.

33. The inverse of a square matrix, if it exists, is unique.

34. The inverse of a matrix can be obtained by applying elementary row operations on the matrix A = IA. In order to use column operations, write A = AI.

35. Either of the two operations—row or column—can be applied. Both cannot be applied simultaneously.

36. For any square matrix A with real number entries, A + A' is a symmetric matrix and A — A' is a skew- symmetric matrix.

Laws of algebra are not applicable to matrices, i.e.

$(A + B)^2 \neq A^2 + 2AB + B^2$

and

$(A + B)(A - B) \neq A^2 - B^2$

Top Formulae

1. An m × n matrix is a square matrix if m = n.

2. $A = [a_{ij}] = [b_{ij}] = B$ if

   (i) A and B are of the same order, (ii) $a_{ij} = b_{ij}$ for all possible values of i and j.

3. $kA = k[a_{ij}]_{m \times n} = [k(a_{ij})]_{m \times n}$

4. $-A = (-1)A$

5. $A - B = A + (-1)B$

6. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$, then $AB = C = [c_{ik}]_{m \times p}$,

   where $c_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{ij}$

7. Elementary operations of a matrix are as follows:
   1. $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
   2. $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$
   3. $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$

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If $A=[a_{ij}]_{m\times n}$, then its transpose $A'\,(A^T)=[a_{ji}]_{n\times m}$ *i.e.* if $A=\begin{pmatrix}2 & 1\end{pmatrix}$ then $A^{T}=\begin{pmatrix}1\\2\end{pmatrix}$.

Also, $(A')'=A$, $(kA)'=kA'$, $(A+B)'=A'+B'$, $(AB)'=B'A'$.

- A is symmetric matrix if $A=A'$ *i.e.* $A'=-A$.
- A is skew – symmetric if $A=-A'$ *i.e.* $A'=-A$.
- A is any matrix, then–

$$A=\frac{1}{2}\left\{\underset{\text{S.M}}{(A+A')}+\underset{\text{Skew, S.M.}}{(A-A')}\right\}=\text{sum of a symmetric and a skew-symmetric matrix.}$$

For *eg* if $A=\begin{pmatrix}2 & 8\\6 & 4\end{pmatrix}$, then $A=\frac{1}{2}\left\{\begin{pmatrix}2 & 7\\7 & 4\end{pmatrix}+\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\right\}$.

A matrix of order m×n is an ordered rectangular array of numbers or functions having 'm' rows and 'n' columns. The matrix $A=[a_{ij}]_{m\times n}$ is given by

$$A=\begin{bmatrix}a_{11} & a_{12} & \dots a_{1n}\\a_{21} & a_{22} & \dots a_{2n}\\a_{m1} & a_{m2} & \dots a_{mn}\end{bmatrix}_{m\times n}$$

- Column matrix : It is of the form $\left[a_{ij}\right]_{m\times 1}$
- Row matrix : It is of the form $\left[a_{ij}\right]_{1\times n}$
- Square matrix : Here, $m=n$ (no. of rows = no. of columns)
- Diagonal matrix : All non-diagonal entries are zero i.e $a_{ij}=0\,\forall i\neq j$
- Scalar matrix : $a_{ij}=0, i\neq j$ and $a_{ij}=k$ (Scalar), $i=j$
- Identity matrix : $a_{ij}=0, i\neq j$ and $a_{ij}=1, i=j$
- Zero matrix : All entries are zero.

Types of Matrix

Definition and its types

$A=\left[a_{ij}\right]=\left[b_{ij}\right]=B$ if, A and B are of same order and $a_{ij}=b_{ij}\ \forall\ i$ and j.

Elementary operations on a matrix

Matrices

Equality of Two Matrix

$R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$

$R_i \to kR_i$ or $C_i \to kC_i$

$R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Operations on matrices

If A, B are two matrices of same order, then $A+B=\left[a_{ij}+b_{ij}\right]$ The addition of A and B follows: $A+B=B+A, (A+B)+C=A+(B+C), -A=(-1)A$, $k(A+B)=kA+kB$, k is scalar and $(k+I)A=kA+IA$, k and I are constants.

If A, B are square matrices such that $AB=BA=I$ then $B=A^{-1}$ i.e., A is the inverse of B and vice-versa.

Inverse of a square matrix, if it exists, is unique

For eg: If $A=\begin{pmatrix}1 & 2\\3 & 4\\5 & 6\end{pmatrix}$, then after $R_1 \leftrightarrow R$, A becomes $\begin{pmatrix}3 & 4\\1 & 2\\5 & 6\end{pmatrix}$

If A and B are invertible matrices of the same order, then $(AB)^{-1}=B^{-1}A^{-1}$. By elementary transformations, we can convert $A=IA$ to $I\,A^{-1}A$. This is one process of finding the inverse of a given square matrix A.

Addition

- If $A=\begin{pmatrix}2 & 3\\2 & 4\end{pmatrix}, B=\begin{pmatrix}-3 & 2\\-4 & 5\end{pmatrix}$ then $A+B=\begin{pmatrix}-1 & 5\\-2 & 9\end{pmatrix}$
- If $A=\begin{pmatrix}2 & 3\end{pmatrix}_{1\times 2}, B=\begin{pmatrix}4\\5\end{pmatrix}_{2\times 1}$, then $AB=\begin{pmatrix}2\times4+3\times5\end{pmatrix}=\begin{pmatrix}2 & 3\end{pmatrix}_{1\times 1}$

Multiplication

If $A=[a_{ij}]_{m\times n}$ and $B=[b_{jk}]_{n\times p}$, then $AB=C=[C_{ik}]_{m\times p}$, $[C_{jk}]=\sum_{j=1}^{n}a_{ij}b_{jk}$. Also, $A(BC)=(AB)C$, $A(B+C)=AB+AC$ and $(A+B)C=AC+BC$, but $AB\neq BA$ (always).

Important Questions

Multiple Choice questions-

1. If A = $[a_{ij}]_{m \times n}$ is a square matrix, if:

(a) m < n

(b) m > n

(c) m = n

(d) None of these.

2. Which of the given values of x and y make the following pair of matrices equal:

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(a) $x = -\frac{1}{3}, y = 7$

(b) Not possible to find

(c) $y = 7, x = -\frac{2}{3}$

(d) $x = -\frac{1}{3}, y = -\frac{2}{3}$

3. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

(a) 27

(b) 18

(c) 81

(d) 512.

Assume X, Y, Z, W and P are matrices of order 2 × n, 3 × 1, 2 × p, n × 3 and p × k respectively. Now answer the following (4-5):

4. The restrictions on n, k and p so that PY + WY will be defined are

(a) k = 3, p = n

(b) k is arbitrary, p = 2

(c) p is arbitrary

(d) k = 2, p = 3.

5. If n =p, then the order of the matrix 7X - 5Z is:

(a) p × 2

(b) 2 × n

(c) n × 3

(d) p × n.

6. If A, B are symmetric matrices of same order, then AB – BA is a

(a) Skew-symmetric matrix

(b) Symmetric matrix

(c) Zero matrix

(d) Identity matrix.

7.

If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then A + A' = I, the value of α is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) π

(d) $\frac{3\pi}{2}$

8. Matrices A and B will be inverse of each other only if:

(a) AB = BA

(b) AB – BA = O

(c) AB = O, BA = I

(d) AB = BA = I.

9.

If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(a) $1 + \alpha^2 + ß\gamma = 0$

(b) $1 - \alpha^2 + ß\gamma = 0$

(c) $1 - \alpha^2 - ß\gamma = 0$

(d) $1 + \alpha^2 - ß\gamma = 0$

10. If a matrix is both symmetric and skew- symmetric matrix, then:

(a) A is a diagonal matrix

(b) A is a zero matrix

(c) A is a square matrix

(d) None of these.

Very Short Questions:

1. If a matrix has 8 elements, what are the possible orders it can have.
2. Identity matrix of orders n is denoted by.
3. Define square matrix
4. The no. of all possible metrics of order 3 × 3 with each entry 0 or 1 is
5. Write (1) a33, a12 (ii) what is its order

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

6. Two matrices A = a_{ij} and B = b_{ij} are said to be equal if
7. Define Diagonal matrix.
8. Every diagonal element of a skew symmetric matrix is
9. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$ Find α

10. $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$ Find $A+A'$

Short Questions:

1. Write the element a_{23} of a 3 x 3 matrix A = $[a_{ij}]$ whose elements atj are given by: $\frac{|i-j|}{2}$

2. For what value of x is

$$[1 \quad 2 \quad 1]\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=0$$? (C.B.S.E. 2019(C))

3. Find a matrix A such that 2A – 3B + 5C = 0,

Where B = $\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}$ and C = $\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$

4. If A = $\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & -\cos\alpha\end{pmatrix}$, then for what value of 'α' is A an identity matrix?

5. Find the values of x, y, z and t, if:

$$2\begin{bmatrix}x & z\\ y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$$

6. If A = $\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, then find $(A^2 - 5A)$. (CBSE 2019)

7. If A = $\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ and I = $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find k so that $A^2 = 5A + b\,kI$

8. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that AB – BA is a skew symmetric matrix. (A.I.C.B.S.E. 2019)

Long Questions:

1. Find the values of a, b, c and d from the following equation:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$ (N.C.E.R.T)

2. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}$ = A + $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ then find the matrix A. (C.B.S.E. 2013)

3. If A = = $\begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$ find the matrix C such that A + B + C is a zero matrix.

4. If A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ B = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ then find the matrix 'X', of order 3 x 2, such that 2A + 3X = 5B. (N.C.E.R.T.)

Assertion and Reason Questions:

1. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion(A): $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.

Reason (R): A matrix A=[a$_{ij}$] is an identity matrix if $a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$.

2. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion (A): Matrix $\begin{bmatrix}1\\5\\2\end{bmatrix}$ is a column matrix.

Reason(R): A matrix of order m × 1 is called a column matrix.

Case Study Questions:

1. Three shopkeepers A, B and C go to a store to buy stationary. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹ 40, a pen costs ₹ 12 and a pencil costs ₹ 3.

Based on the above information, answer the following questions.

(i) The number of items purchased by shopkeepers A, B and C represented in matrix form as:

a. $$\begin{array}{c} \begin{array}{ccc}\text{Notebooks} & \text{Pens} & \text{Pencils}\end{array} \\ \begin{bmatrix}144 & 60 & 72\\ 120 & 720 & 84\\ 132 & 156 & 96\end{bmatrix}\begin{matrix}A\\ B\\ C\end{matrix}\end{array}$$

b. $$\begin{array}{c} \begin{array}{ccc}\text{Notebooks} & \text{Pens} & \text{Pencils}\end{array} \\ \begin{bmatrix}144 & 72 & 60\\ 120 & 84 & 72\\ 132 & 156 & 96\end{bmatrix}\begin{matrix}A\\ B\\ C\end{matrix}\end{array}$$

c. $$\begin{array}{c} \begin{array}{ccc}\text{Notebooks} & \text{Pens} & \text{Pencils}\end{array} \\ \begin{bmatrix}144 & 72 & 72\\ 120 & 156 & 84\\ 132 & 84 & 96\end{bmatrix}\begin{matrix}A\\ B\\ C\end{matrix}\end{array}$$

d. $$\begin{array}{c} \begin{array}{ccc}\text{Notebooks} & \text{Pens} & \text{Pencils}\end{array} \\ \begin{bmatrix}144 & 60 & 60\\ 120 & 84 & 72\\ 132 & 156 & 96\end{bmatrix}\begin{matrix}A\\ B\\ C\end{matrix}\end{array}$$

(ii) If Y represents the matrix formed by the cost of each item, then XY equals.

a. $\begin{bmatrix} 5741 \\ 6780 \\ 8040 \end{bmatrix}$

b. $\begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$

c. $\begin{bmatrix} 5916 \\ 6696 \\ 7440 \end{bmatrix}$

d. $\begin{bmatrix} 6740 \\ 5740 \\ 8140 \end{bmatrix}$

(iii) Bill of A is equal to:

a. ₹ 6740

b. ₹ 8140

c. ₹ 5740

d. ₹ 6696

(iv) If $A^2 = A$, then $(A + 1)^3 - 7A =$

a. A

b. A - I

c. I

d. A + I

(v)If A and B are 3 × 3 matrices such that $A^2 - B^2 = (A - B)(A + B)$, then

a. Either A or B is zero matrix.

b. Either A or B is unit matrix.

c. A = B

d. AB = BA

2. Consider 2 families A and B. Suppose there are 4 men,4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommend daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for a children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

Based on the above information, answer the following questions.

(i) The requirement of calories and proteins for each person in matrix form can be represented as:

a.

$$\begin{array}{ll} & \begin{array}{cc}\text{Calorise} & \text{Proteins}\end{array} \\ \begin{array}{l}\text{Man}\\ \text{Woman}\\ \text{Children}\end{array} & \begin{bmatrix}2400 & 45\\ 1900 & 55\\ 1800 & 33\end{bmatrix}\end{array}$$

b.

$$\begin{array}{ll} & \begin{array}{cc}\text{Calorise} & \text{Proteins}\end{array} \\ \begin{array}{l}\text{Man}\\ \text{Woman}\\ \text{Children}\end{array} & \begin{bmatrix}1900 & 55\\ 2400 & 45\\ 1800 & 33\end{bmatrix}\end{array}$$

c.

$$\begin{array}{ll} & \begin{array}{cc}\text{Calorise} & \text{Proteins}\end{array} \\ \begin{array}{l}\text{Man}\\ \text{Woman}\\ \text{Children}\end{array} & \begin{bmatrix}1800 & 33\\ 1900 & 55\\ 2400 & 45\end{bmatrix}\end{array}$$

d.

$$\begin{array}{ll} & \begin{array}{cc}\text{Calorise} & \text{Proteins}\end{array} \\ \begin{array}{l}\text{Man}\\ \text{Woman}\\ \text{Children}\end{array} & \begin{bmatrix}2400 & 33\\ 1900 & 55\\ 1800 & 45\end{bmatrix}\end{array}$$

(ii) Requirement of calories of family A is:

a. 24000
b. 24400
c. 15000
d. 15800

(iii) Requirement of proteins for family B is:

a. 560 grams
b. 332 grams
c. 266 grams
d. 300 grams

(iv) If A and Bare two matrices such that AB = B and BA = A, then $A^2 + B^2$ equals.

a. 2AB
b. 2BA
c. A + B
d. AB

(v)If $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{n\times p}$ and $C = (c_{ij})_{p\times q}$ then the product (BC) A is possible only when.

a. m = q
b. n = q
c. p = q
d. m = p

Answer Key-

Multiple Choice questions-

1. Answer: (c) m = n
2. Answer: (b) Not possible to find
3. Answer: (d) 512.
4. Answer: (a) k = 3, p = n
5. Answer: (b) 2 × n
6. Answer: (a) Skew-symmetric matrix
7. Answer: (a) $\frac{\pi}{6}$
8. Answer: (d) AB = BA = I.
9. Answer: (c) $1 - \alpha^2 - ß\gamma = 0$
10. Answer: (b) A is a zero matrix

Very Short Answer:

1. Solution:

$1\times 8,\ 8\times 1,\ 4\times 2,\ 2\times 4,$

2. Solution: I_n

3. Solution: A matrix in which the no. of rows are equal to no. of columns i.e. m = n

4. Solution: $512=2^9$

5. Solution:

(i) $a_{33} = 9$, $a_{12} = 4$

(ii) 3 × 3

6. Solution: They are of the same order.

7. Solution: A square matrix in which every non – diagonal element is zero is called diagonal matrix.

8. Solution: Zero.

9. Solution:

$$A + A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix}$$

$$A + A' = I(Given)$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2\cos\alpha = 1$$

$$\cos\alpha = \frac{1}{2}$$

$$\cos\alpha = \cos\frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3}$$

10. Solution:

$$A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

Short Answer:

1. Solution:

We have $[a_{ij}] = \frac{|i-j|}{2}$

$\therefore a_{23} = \frac{|2-3|}{2} = \frac{|-1|}{2} = \frac{1}{2}$

2. Solution:

We have

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$[1 + 4 + 1 \; 2 + 0 + 0 \; 0 + 2 + 2] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$[6 \; 2 \; 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$\Rightarrow [0 + 4 + 4x] = 0$

$\Rightarrow [4 + 4x] = [0]$

$\Rightarrow 4 + 4x = 0.$

Hence, $x = -1$.

3. Solution:

Here, 2A - 3B + 5C = 0

⇒ 2A = 3B - 5C

$$\Rightarrow 2A = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} -10 & 0 & 10 \\ -35 & -5 & -30 \end{bmatrix}$$

$$= \begin{bmatrix} -6-10 & 6+0 & 0+10 \\ 9-35 & 3-5 & 12-30 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}.$$

Hence, $A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}.$

4. Solution:

Here A = $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix}$

Now A = I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ when

cos α = 1 and sin α = 0.

Hence, α = 0.

5. Solution:

We have:

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$\Rightarrow 2x + 3 = 9$ .... (1)

$2z - 3 = 15$ .... (2)

$2y = 12$ .... (3)

$2t + 6 = 18$ ………… (4)

From (1), $\Rightarrow 2x = 9 - 3$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3.$

From (3) $2y = 12$

$\Rightarrow y = 6.$

From (2), $\Rightarrow 2z - 3 = 15$

$\Rightarrow 2z = 18$

$\Rightarrow z = 9.$

From (4), $2t + 6 = 18$

$\Rightarrow 2t = 12$

$\Rightarrow t = 6.$

Hence, $x = 3$, $y = 6$, $z = 9$ and $t = 6$.

6. Solution:

We have $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Then $A^2 = AA$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}.$$

$$\therefore A^2 - 5A = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}.$$

7. Solution:

We have : $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

$$\therefore A^2 = AA = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \quad \ldots (1)$$

Also, $5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \quad \ldots(2)$

and $kI = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \quad \ldots(3)$

$\therefore A^2 = 5A + kI$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

[Using (1), (2) & (3)]

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15+k & 5 \\ -5 & 10+k \end{bmatrix}$$

$\Rightarrow$ 8 = 15 + k and 3 = 10 + k

$\Rightarrow$ k = -1 and k = -7.

Hence, k – (-7).

8. Solution:

Since A and B are symmetric matrices,

∴ A’ = A and B’ = B ...(1)

Now, (AB – BA)’= (AB)’ – (BA)’

= B’A’ – A’B’

= BA – AB [Using (1)]

= – (AB – BA).

Hence, AB – BA is a skew-symmetric matrix.

Long Answer:

1. Solution:

We have

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Comparing the corresponding elements of two given matrices, we get:

2a + b = 4 ...(1)

a-2b = – 3 ...(2)

5c-d = 11 ...(3)

$4c + 3d = 24$ ...(4)

Solving (1) and (2):

From (1),

$b = 4 - 2a$ ...(5)

Putting in (2), $a - 2(4 - 2a) = -3$

$\Rightarrow a - 8 + 4a = -3$

$\Rightarrow 5a = 5$

$\Rightarrow a = 1$.

Putting in (5),

$b = 4 - 2(1) = 4 - 2 = 2$.

Solving (3) and (4):

From (3),

$d = 5c - 11$ ...(6)

Putting in (4),

$4c + 3(5c - 11) = 24$

$\Rightarrow 4c + 15c - 33 = 24$

$\Rightarrow 19c = 57$

$\Rightarrow c = 3$.

Putting in (6),

$d = 5(3) - 11 = 15 - 11 = 4$.

Hence, $a = 1$, $b = 2$, $c = 3$ and $d = 4$.

2. Solution:

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$.

Then $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+1 & a_{12}+2 & a_{13}-1 \\ a_{21}+0 & a_{22}+4 & a_{23}+9 \end{bmatrix}.$$

Comparing:

9 = a_{11} + 1 – 1 = a_{12} + 2,

4 = 1_{13} – 1, -2 = a_{21}

1 = a_{22} + 4, and 3 = a_{23} + 9

a_{11} = 8, a_{12} = – 3,

a_{13} = 5, a_{21} = -2

a_{22} = – 3, and a_{23} = – 6.

Hence, $A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

3. Solution:

Let $C=\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$.

Then A + B + C = O

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}+\begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}+\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+6 & 2+2 \\ -3+1 & 1+3 \\ 4+0 & 0+4 \end{bmatrix}+\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+6+c_{11} & 2+2+c_{12} \\ -3+1+c_{21} & 1+3+c_{22} \\ 4+0+c_{31} & 0+4+c_{32} \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8+c_{11} & 4+c_{12} \\ -2+c_{21} & 4+c_{22} \\ 4+c_{31} & 4+c_{32} \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Comparing:

$8 + c_{11} = 0 \Rightarrow c_{11} = -8$,

$4 + C_{12} = 0 \Rightarrow C_{12} = -4$,

$-2 + C_{21} = 0 \Rightarrow C_{21} = 2$

$4 + C_{22} = 0 \Rightarrow C_{22} =- 4$,

$4 + c_{31} = 0 \Rightarrow C_{31} = -4$

and $4 + c_{32} = 0 \Rightarrow C_{32} = -4$.

Hence, $C = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$

4. Solution:

We have: 2A + 3X = 5B

⇒ 2A + 3X-2A = 5B-2A

⇒ 2A-2A + 3X = 5B-2A

⇒ (2A – 2A) + 3X = 5B – 2A

⇒ O + 3X = 5B – 2A

[∵ – 2A is the inverse of2A]

⇒ 3X = 5B – 2A.

[∵ O is the additive identity]

Hence, X = $\frac{1}{3}$ (5B – 2A)

$$=\frac{1}{3}\left(5\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}-2\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}\right)$$

$$=\frac{1}{3}\left(\begin{bmatrix}10 & -10\\ 20 & 10\\ -25 & 5\end{bmatrix}+\begin{bmatrix}-16 & 0\\ -8 & 4\\ -6 & -12\end{bmatrix}\right)$$

$$=\frac{1}{3}\begin{bmatrix}10-16 & -10+0\\ 20-8 & 10+4\\ -25-6 & 5-12\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}-6 & -10\\ 12 & 14\\ -31 & -7\end{bmatrix}=\begin{bmatrix}-2 & -10/3\\ 4 & 14/3\\ -31/3 & -7/3\end{bmatrix}$$

Assertion and Reason Answers:

1. (d) A is false and R is true.

Solution:

We know that, $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ is an indentity matrix

∴ Given Assertion [A] is false We know that for identity matrix $a_{ij} = 1, ifi = j$ and $a_{ij} = 0, ifi \neq j$

∴ Given Reason (R) is true Hence option (d) is the correct answer.

2. a) Both A and R are true and R is the correct explanation of A.

Solution:

We know that order of column matrix is always $m \times 1$

$\therefore \begin{bmatrix}1\\5\\2\end{bmatrix}$ is column matrix.

⇒ Assertion (A) is true Also Reason (R) is true and is correct explanation of A. Hence option (a) is the correct answer.

Case Study Answers:

1. Answer :

i. (a)
$$\begin{array}{cc} \begin{array}{ccc}\text{Notebooks} & \text{Pens} & \text{Pencils}\end{array} & \\ \begin{bmatrix}144 & 60 & 72\\ 120 & 720 & 84\\ 132 & 156 & 96\end{bmatrix} & \begin{array}{c}A\\ B\\ C\end{array}\end{array}$$

Solution:

$$X = \begin{array}{cc} \begin{array}{ccc}\text{Notebooks} & \text{Pens} & \text{Pencils}\end{array} & \\ \begin{bmatrix}144 & 60 & 72\\ 120 & 720 & 84\\ 132 & 156 & 96\end{bmatrix} & \begin{array}{c}A\\ B\\ C\end{array}\end{array}$$

ii. (b) $\begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$

Solution:

Since, $Y = \left[\begin{array}{c|c} 40 & \text{Notebook} \\ 12 & \text{Pens} \\ 3 & \text{Pencil} \end{array}\right]$

$$\therefore \quad XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5760 + 720 + 216 \\ 4800 + 864 + 252 \\ 5280 + 1872 + 288 \end{bmatrix} = \begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$$

iii. (d) ₹ 6696

Solution:

Bill of A is ₹ 6696.

iv. (c) I

Solution:

$(A + I)^2 = A^2 + 2A + I = 3A + I$

$\Rightarrow (A + 1)^3 = (3A + I)(A + I)$

$= 3A^2 + 4A + I = 7A + I$

∴∴ $(A + I)^3 - 7A = I$

v. (d) AB = BA

Solution:

$A^2 - B^2 = (A - B)(A + B) = A^2 + AB - BA - B^2$

∴∴ AB = BA

2. Answer :

i. (a)

$$\begin{array}{c} & \text{Calorise} & \text{Proteins} \\ \begin{array}{c}\text{Man}\\ \text{Woman}\\ \text{Children}\end{array} & \left[\begin{array}{c}2400\\1900\\1800\end{array}\right. & \left.\begin{array}{c}45\\55\\33\end{array}\right]\end{array}$$

Solution:

Let F be the matrix representing the number of family members and

R be the matrix representing the requirement of calories and proteins

for each person. Then

$$F = \begin{array}{c} & \text{Men} & \text{Women} & \text{Children} \\ \begin{array}{c}\text{Family A}\\ \text{Family B}\end{array} & \left[\begin{array}{c}4\\2\end{array}\right. & \begin{array}{c}4\\2\end{array} & \left.\begin{array}{c}4\\2\end{array}\right]\end{array}$$

$$R = \begin{array}{c} & \text{Calorise} & \text{Proteins} \\ \begin{array}{c}\text{Man}\\ \text{Woman}\\ \text{Children}\end{array} & \left[\begin{array}{c}2400\\1900\\1800\end{array}\right. & \left.\begin{array}{c}45\\55\\33\end{array}\right]\end{array}$$

ii. (b) 24400

Solution:

The requirement of calories and proteins for each ofthe two families is given by the product matrix FR.

$$FR = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 4(2400+1900+1800) & 4(45+55+33) \\ 2(2400+1900+1800) & 2(45+55+33) \end{bmatrix}$$

$$FR = \begin{array}{cc} \begin{array}{cc}\text{Calories} & \text{Proteins}\end{array} & \\ \begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix} & \begin{array}{c}\text{Family A}\\ \text{Family B}\end{array}\end{array}$$

iii. (c) 266 grams

iv. (c) A + B

Solution:

Since, AB = B … (i)

BA = A … (ii)

$\therefore A^2 + B^2 = A \times A + B \times B$

= A(BA) + B(AB)

= (AB)A + (BA)B

= BA + AB

= A + B

v. (a) m = q

Solution:

$$A = (a_{ij})_{m\times n},\quad B = (b_{ij})_{n\times p},\quad C = (c_{ij})_{p\times q}$$

$$BC = (b_{ij})_{n\times p} \times (c_{ij})_{p\times q} = (d_{ij})_{n\times q}$$

$$(BC)A = (d_{ij})_{n\times q} \times (a_{ij})_{m\times n}$$

Hence, (BC)A is possible only when m = q