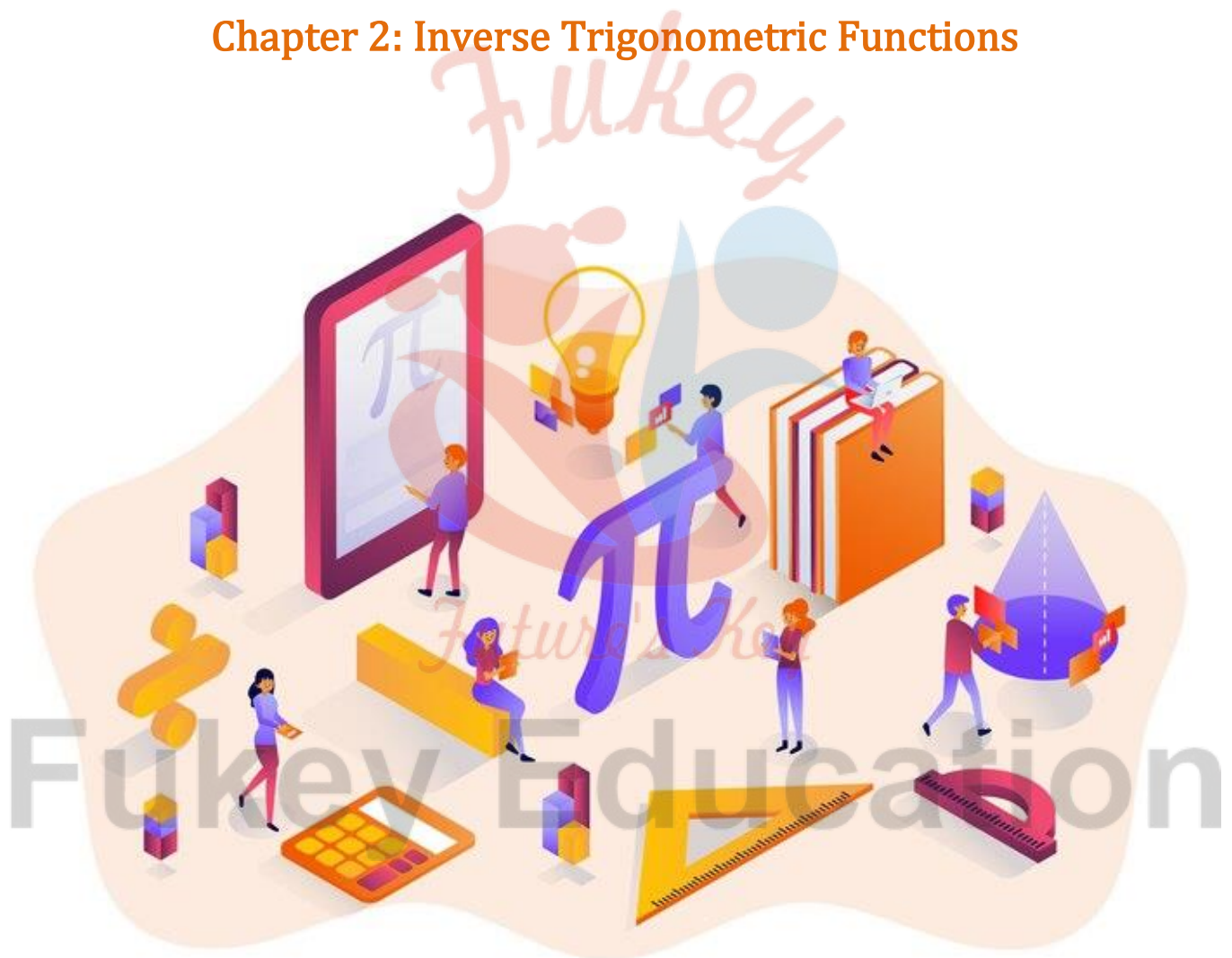


MATHEMATICS

Chapter 2: Inverse Trigonometric Functions



INVERSE TRIGONOMETRIC FUNCTIONS

1. The domains and ranges (principal value branches) of inverse trigonometric functions are given in the following table:

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1}x$	$\mathbb{R} - [-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$\mathbb{R} - [-1, 1]$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1}x$	\mathbb{R}	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cot^{-1}x$	\mathbb{R}	$[0, \pi]$

2. $\sin^{-1}x$ should not be confused with $(\sin^{-1}x)$. In fact, $(\sin x)^{-1} = \frac{1}{\sin x}$ And similarly for other trigonometric functions.
3. The value of an inverse trigonometric functions which lies in its principal value branch is called the principal value of that inverse trigonometric functions.
4. For suitable values of domain, we have

- $y = \sin^{-1}x \Rightarrow x = \sin y$

$$\bullet x = \sin y \Rightarrow y = \sin^{-1} x$$

$$\bullet \sin (\sin^{-1} x) = x$$

$$\bullet \sin^{-1} (\sin x) = x$$

$$\bullet \sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} x$$

$$\bullet \cos^{-1} (-x) = \pi - \cos^{-1} x$$

$$\bullet \cos^{-1} \frac{1}{x} = \operatorname{sec}^{-1} x$$

$$\bullet \cot^{-1}(-x) = \pi - \cot^{-1} x$$

$$\bullet \tan^{-1} \frac{1}{x} = \cot^{-1} x$$

$$\cot^{-1} \frac{1}{x} = \tan^{-1} x$$

$$\operatorname{cosec}^{-1} \frac{1}{x} = \sin^{-1} x$$

$$\bullet \sec^{-1}(-x) = \pi - \sec^{-1} x$$

$$\sec^{-1} \frac{1}{x} = \cos^{-1} x$$

$$\bullet \sin^{-1} (-x) = -\sin^{-1} x$$



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$$\bullet \tan^{-1}(-x) = -\tan^{-1}x$$

$$\bullet \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

$$\bullet \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$$

$$\bullet \operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$$

$$\bullet \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\bullet \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$$\bullet \tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$2\cos^{-1}x = \cos^{-1}\left(2x^2 - 1\right)$$

$$\bullet 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$$

$$3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right)$$

$$3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

5. Conversion:

- $\sin^{-1} x = \cos^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{x}{\sqrt{1-x^2}} = \cot^{-1} \frac{\sqrt{1-x^2}}{x} = \sec^{-1} \frac{1}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1} \frac{1}{x}$
- $\cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{\sqrt{1-x^2}}{x} = \cot^{-1} \frac{x}{\sqrt{1-x^2}} = \sec^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} \frac{1}{\sqrt{1-x^2}}$
- $\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}} = \cos^{-1} \frac{1}{\sqrt{1+x^2}} = \sec^{-1} \sqrt{1+x^2} = \operatorname{cosec}^{-1} \frac{\sqrt{1+x^2}}{x} = \cot^{-1} \frac{1}{x}$
- $\cot^{-1} x = \sin^{-1} \frac{1}{\sqrt{1+x^2}} = \cos^{-1} \frac{x}{\sqrt{1+x^2}} = \sec^{-1} \frac{1}{x} = \sec^{-1} \frac{\sqrt{1+x^2}}{x} = \operatorname{cosec}^{-1} \sqrt{1+x^2}$
- $\sec^{-1} x = \tan^{-1} \frac{\sqrt{x^2-1}}{1} = \cot^{-1} \frac{1}{\sqrt{x^2-1}} = \sin^{-1} \frac{\sqrt{x^2-1}}{x} = \cos^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} \frac{x}{\sqrt{x^2-1}}$
- $\operatorname{cosec}^{-1} x = \sin^{-1} \frac{1}{x} = \tan^{-1} \frac{1}{\sqrt{x^2-1}} = \cot^{-1} \sqrt{x^2-1} = \sec^{-1} \frac{x}{\sqrt{x^2-1}} = \cos^{-1} \frac{\sqrt{x^2-1}}{x}$

6. Some other properties of Inverse Trigonometric Function:

- $\tan^{-1} \frac{x}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a}$
- $\cot^{-1} \frac{x}{\sqrt{a^2-x^2}} = \cos^{-1} \frac{x}{a}$
- $\tan^{-1} \frac{a}{\sqrt{x^2-a^2}} = \operatorname{cosec}^{-1} \frac{x}{a}$
- $\cot^{-1} \frac{a}{\sqrt{x^2-a^2}} = \sec^{-1} \frac{x}{a}$

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- (i) $y = \sin^{-1}x$. Domain = $[-1,1]$, Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(ii) $y = \cos^{-1}x$. Domain = $[-1,1]$ Range = $[0, \pi]$
(iii) $y = \operatorname{cosec}^{-1}x$. Domain = $R - (-1,1)$, Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
(iv) $y = \sec^{-1}x$. Domain = $R - (-1,1)$, Range = $[0, \pi] - \left\{\frac{\pi}{2}\right\}$
(v) $y = \tan^{-1}x$. Domain = R , Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(vi) $y = \cot^{-1}x$. Domain = R , Range = $(0, \pi)$.

- (I) $\sin : R \rightarrow [-1,1]$
(ii) $\cos : R \rightarrow [-1,1]$
(iii) $\tan : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \rightarrow R$
(iv) $\cot : R - \{x : x = h\pi, n \in Z\} \rightarrow R$
(v) $\sec : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \rightarrow R - (-1,1)$
(vi) $\operatorname{cosec} : R - \{x : x = h\pi, n \in Z\} \rightarrow R - (-1,1)$

Domain and range of inverse trigonometric functions

Trigonometric functions

Some important relations

$\sin^{-1}x \neq (\sin x)^{-1}$, $(\sin x)^{-1} = \frac{1}{\sin x}$ and same for other trigonometric functions.

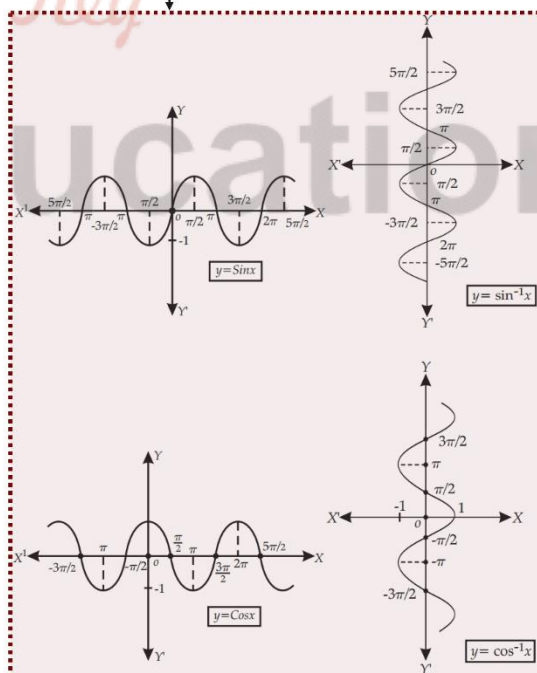
Inverse Trigonometric Functions

Graphs of trigonometric functions and inverse trigonometric functions

The range of an inverse trigonometric function is the principal value branch and those values which lies in the principal value branch is called the principal value of that inverse trigonometric functions.

Principal Value branch and principal value

- (i) $y = \sin^{-1}x \Rightarrow x = \sin y$ (ii) $x = \sin y \Rightarrow y = \sin^{-1}x$
(iii) $\sin(\sin^{-1}x) = x$ (iv) $\sin^{-1}(\sin x) = x$
(v) $\sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1}x$ (vi) $\cos^{-1}(-x) = \pi - \cos^{-1}x$
(vii) $\cos^{-1} \frac{1}{x} = \sec^{-1}x$ (viii) $\cot^{-1}(-x) = \pi - \cot^{-1}x$
(ix) $\tan^{-1} \frac{1}{x} = \cot^{-1}x$ (x) $\sec^{-1}(-x) = \pi - \sec^{-1}x$
(xi) $\sin^{-1}(-x) = -\sin^{-1}x$ (xii) $\tan^{-1}(-x) = -\tan^{-1}x$
(xiii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ (xiv) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$
(xv) $\tan^{-1}x + \tan^{-1}y = \tan^{-1} \frac{x+y}{1-xy}$ (xvi) $2 \tan^{-1}x = \tan^{-1} \frac{2x}{1-x^2}$
(xvii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1} \frac{x-y}{1+xy}$ (xviii) $2 \tan^{-1}x = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2}$
- For eg : to find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$,
 $\Rightarrow \sin y = \frac{1}{\sqrt{2}}$ The range or the principal value branch of \sin^{-1} is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
and $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ So, the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$



Important Questions

Multiple Choice questions-

1. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is

- (a) $\frac{2\pi}{3}$
- (b) $\frac{\pi}{3}$
- (c) $\frac{\pi}{2}$
- (d) π

2. $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is equal to:

- (a) π
- (b) $-\frac{\pi}{3}$
- (c) $\frac{\pi}{3}$
- (d) $\frac{2\pi}{3}$

3. $\cos^{-1}(\cos \frac{7\pi}{6})$ is equal to

- (a) $\frac{7\pi}{6}$
- (b) $-\frac{5\pi}{6}$
- (c) $\frac{\pi}{3}$
- (d) $\frac{\pi}{6}$

4. $\sin(\frac{\pi}{3} - \sin^{-1}(-\frac{1}{2}))$ is equal to

- (a) $\frac{1}{2}$
- (b) $\frac{1}{3}$
- (c) $\frac{1}{4}$
- (d) 1

5. $\tan^{-1} \sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to

- (a) π
- (b) $-\frac{\pi}{2}$
- (c) 0
- (d) $2\sqrt{3}$

6. $\sin(\tan^{-1} x), |x| < 1$, is equal to

- (a) $\frac{x}{\sqrt{1-x^2}}$
- (b) $\frac{1}{\sqrt{1-x^2}}$
- (c) $\frac{x}{\sqrt{1+x^2}}$
- (d) $\frac{x}{\sqrt{1+x^2}}$

7. $\sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$, then x is equal to

- (a) $0, \frac{1}{2}$
- (b) $1, \frac{1}{2}$
- (c) 0
- (d) $\frac{1}{2}$

8. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ is equal to

- (a) $\frac{\pi}{2}$
- (b) $\frac{\pi}{3}$
- (c) $\frac{\pi}{4}$
- (d) $-\frac{3\pi}{4}$

9. The value of $\sin^{-1}(\cos(\frac{43\pi}{5}))$ is

- (a) $\frac{3\pi}{5}$
- (b) $\frac{-7\pi}{5}$
- (c) $\frac{\pi}{10}$
- (d) $-\frac{\pi}{10}$

10. The principal value of the expression

$\cos^{-1}[\cos(-680^\circ)]$ is

- (a) $\frac{2\pi}{9}$
- (b) $\frac{-2\pi}{9}$
- (c) $\frac{34\pi}{9}$
- (d) $-\frac{\pi}{9}$

Very Short Questions:

1. Find the principal value of $\sin^{-1}(\frac{1}{2})$

2. What is the principal value of:

$$\cos^{-1}(\cos \frac{2\pi}{3}) + \sin^{-1}(\sin \frac{2\pi}{3})?$$

3. Find the principal value of:

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2). \text{ (A.I.C.B.S.E. 2012)}$$

4. Evaluate: $\tan^{-1}(2 \cos(2 \sin^{-1}(\frac{1}{2})))$

5. Find the value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$. (C.B.S.E. 2018)

6. If $\sin^{-1}(\frac{1}{3}) + \cos^{-1} x = \frac{\pi}{2}$, then find x. (C.B.S.E. 2010C)

7. If $\sec^{-1}(2) + \operatorname{cosec}^{-1}(y) = \frac{\pi}{2}$, then find y.

8. Write the value of $\sin[\frac{\pi}{3} - \sin^{-1}(-\frac{1}{2})]$

9. Prove the following:

$$\cos \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2} \right) = \frac{6}{5\sqrt{13}}$$

10. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, $xy < 1$, then write the value of the $x + y + xy$ (A.I.C.B.S.E. 2014)

Short Questions:

1. Express \sin^{-1}

$$\left(\frac{\sin x + \cos x}{\sqrt{2}} \right)$$

Where $-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form.

2. Prove that:

$$\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65} \quad (\text{A.I.C.B.S.E. 2019; C.B.S.E. 2010})$$

3. Prove that:

$$\sin^{-1} \frac{8}{17} + \cos^{-1} \frac{4}{5} = \cos^{-1} \frac{36}{77} \quad (\text{A.I.C.B.S.E. 2019})$$

4. Solve the following equation:

$$\tan^{-1} \left(\frac{x+1}{x-1} \right) + \tan^{-1} \left(\frac{x-1}{x} \right) = \tan^{-1}(-7) \quad (\text{A.I.C.B.S.E. 2019; C})$$

5. Solve the following equation:

$$2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x), x \neq \frac{\pi}{2} \quad (\text{C.B.S.E. (F) 2012})$$

6. Solve the following equation

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4}\right)$$

(A.I.C.B.S.E. 2013)

7. Prove that

$$3\cos^{-1} x = \cos^{-1}(4x^3 - 3x), x \in \left[\frac{1}{2}, 1 \right]$$

Long Questions:

1. prove that

$$\frac{1}{2} \leq x \leq 1, \text{ then}$$

$$\cos^{-1} x + \cos^{-1} \left[\frac{x}{2} + \frac{\sqrt{3}-3x^2}{2} \right] = \frac{\pi}{3}$$

2. Find the value of:

$$\tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right)$$

3. Prove that:

$$\tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{5} \right) + \tan^{-1} \left(\frac{1}{8} \right) = \frac{\pi}{4}$$

4.

$$2 \sin^{-1} \left(\frac{3}{5} \right) - \tan^{-1} \left(\frac{17}{31} \right) = \frac{\pi}{4}$$

Assertion and Reason Questions:

1. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

- Both A and R are true and R is the correct explanation of A.
- Both A and R are true but R is not the correct explanation of A.
- A is true but R is false.
- A is false and R is true.
- Both A and R are false.

Assertion(A): A relation $R = \{(1, 1), (1, 3), (3, 1), (3, 3), (3, 5)\}$ defined on the set $A = \{1,3,5\}$ is reflexive.

Reason (R): A relation R on the set A is said to be transitive if for. $(a, b) \in R$ and $(b, c) \in R$, we have $(a,c) \in R$.

2. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

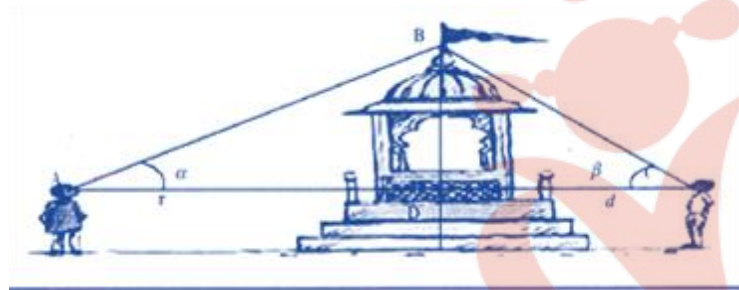
- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false and R is true.
- e) Both A and R are false.

Assertion (A): A relation $R = \{(a,b):|a-b| < 2\}$ defined on the set $A = \{1,2,3,4,5\}$ is reflexive.

Reason(R): A relation R on the set A is said to be reflexive if for $(a,b) \in R$ and $(b,c) \in R$, we have $(a,c) \in R$.

Case Study Questions:

1.



Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively, (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters. Based on the above information answer the following :

- (i) $\angle CAB = \alpha =$
- A) $\sin^{-1} \left(\frac{2}{\sqrt{3}} \right)$
 - B) $\sin^{-1} \left(\frac{1}{2} \right)$
 - C) $\sin^{-1} (2)$
 - D) $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$

(ii) $\angle CAB = \alpha =$

A) $\cos^{-1} \left(\frac{1}{5} \right)$

B) $\cos^{-1} \left(\frac{2}{5} \right)$

C) $\cos^{-1} \left(\frac{\sqrt{3}}{2} \right)$

D) $\cos^{-1} \left(\frac{4}{5} \right)$

(iii) $\angle BCA = \beta =$

A) $\tan^{-1} \left(\frac{1}{2} \right)$

B) $\tan^{-1} (2)$

C) $\tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$

D) $\tan^{-1} (\sqrt{3})$

(iv) $\angle ABC =$

A) $\frac{\pi}{4}$

B) $\frac{\pi}{6}$

C) $\frac{\pi}{2}$

D) $\frac{\pi}{3}$

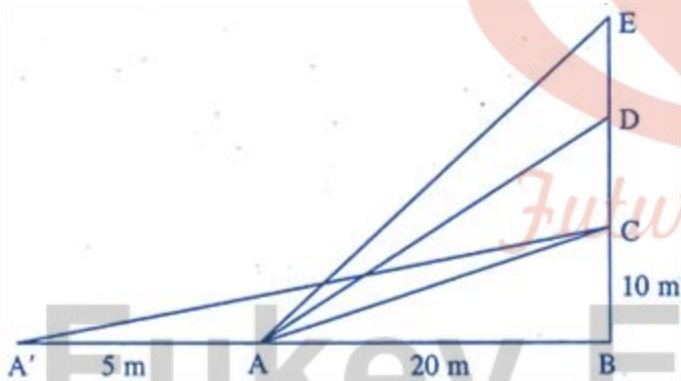


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- (v) Domain and Range of $\cos^{-1}x =$
- A) $(-1, 1), (0, \pi)$
- B) $[-1, 1], (0, \pi)$
- C) $[-1, 1], [0, \pi]$
- D) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is at the height of 10 metres from the ground level.

For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C". The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer. Look at the figure given and based on the above information answer the following :



- (i) Measure of $\angle CAB =$
- A) $\tan^{-1}(2)$
- B) $\tan^{-1}\left(\frac{1}{2}\right)$
- C) $\tan^{-1}(1)$
- D) $\tan^{-1}(3)$

(ii) Measure of $\angle DAB =$

A) $\tan^{-1} \left(\frac{3}{4} \right)$

B) $\tan^{-1} (3)$

C) $\tan^{-1} \left(\frac{4}{3} \right)$

D) $\tan^{-1} (4)$

(iii) Measure of $\angle EAB =$

A) $\tan^{-1} (11)$

B) $\tan^{-1} 3$

C) $\tan^{-1} \left(\frac{2}{11} \right)$

D) $\tan^{-1} \left(\frac{11}{2} \right)$

(iv) A' is another viewer standing on the same line of observation across the road.

If the width of the road is 5 meters, then the difference between $\angle CAB$ and $\angle CA'B$ is

A) $\tan^{-1} (1/2)$

B) $\tan^{-1} \left(\frac{1}{12} \right)$

C) $\tan^{-1} \left(\frac{2}{5} \right)$

D) $\tan^{-1} \left(\frac{11}{21} \right)$

(v) Domain and Range of $\tan^{-1} x =$

A) R^+ , $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

B) R^- , $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C) R , $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

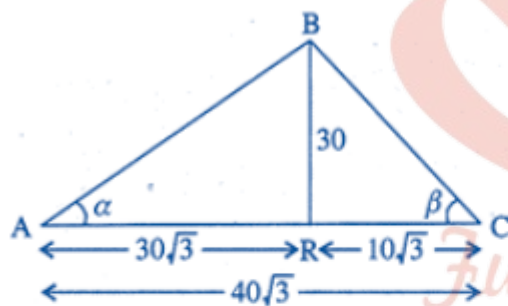
D) R , $\left(0, \frac{\pi}{2}\right)$

Case Study Answers:

1.

(i) (b)

Solution:



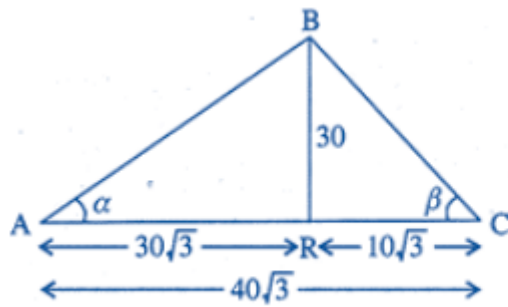
$$AB^2 = (AD)^2 + (BD)^2 = (30\sqrt{3})^2 + (30)^2$$

$$= 2700 + 900 = 3600 \Rightarrow AB = 60 \text{ m}$$

$$\sin \alpha = \frac{BD}{AB} = \frac{30}{60} = \frac{1}{2} \Rightarrow \alpha = \sin^{-1} \left(\frac{1}{2} \right)$$

(ii) (c)

Solution:



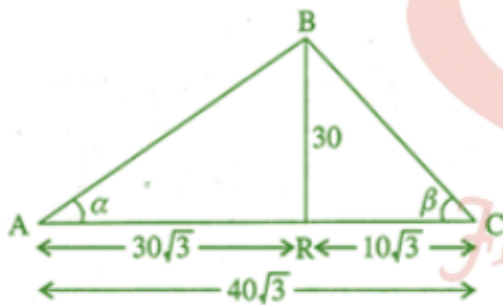
$$AB^2 = (AD)^2 + (BD)^2 = (30\sqrt{3})^2 + (30)^2$$

$$= 2700 + 900 = 3600 \Rightarrow AB = 60 \text{ m}$$

$$\cos \alpha = \frac{AD}{AB} = \frac{30\sqrt{3}}{60} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \tan^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

(iii) (d)

Solution:



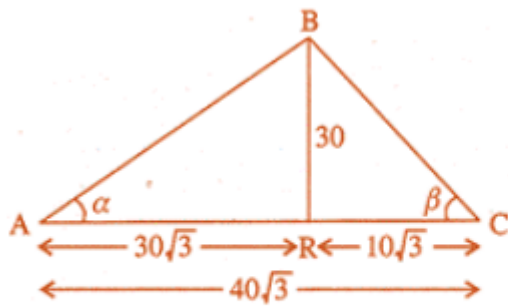
$$AB^2 = (AD)^2 + (BD)^2 = (30\sqrt{3})^2 + (30)^2$$

$$= 2700 + 900 = 3600 \Rightarrow AB = 60 \text{ m}$$

$$\tan \beta = \frac{BD}{DC} = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \beta = \tan^{-1}(\sqrt{3})$$

(iv) (c)

Solution:



$$AB^2 = (AD)^2 + (BD)^2 = (30\sqrt{3})^2 + (30)^2$$

$$= 2700 + 900 = 3600 \Rightarrow AB = 60 \text{ m}$$

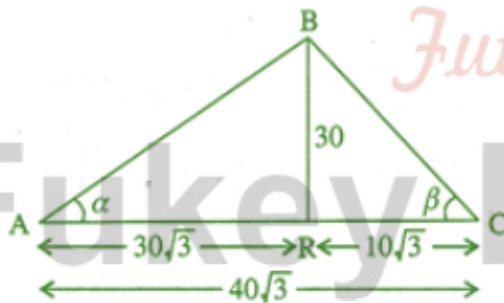
$$\sin \alpha = \frac{1}{2} \Rightarrow \alpha = 30^\circ; \tan \beta = \sqrt{3}$$

$$\Rightarrow \beta = 60^\circ \therefore \angle ABC + \alpha + \beta = 180^\circ$$

$$\Rightarrow \angle ABC + 30^\circ + 60^\circ = 90^\circ = \frac{\pi}{2}$$

(v) (c)

Solution:



$$AB^2 = (AD)^2 + (BD)^2 = (30\sqrt{3})^2 + (30)^2$$

$$= 2700 + 900 = 3600 \Rightarrow AB = 60 \text{ m}$$

$$\text{of } \cos^{-1} x = [-1, 1] \text{ Range of } \cos^{-1} x = [0, \pi]$$

2.

(i) (b)

Solution:

$$\text{Let } \angle BAC = \alpha \text{ and } \angle BA'C = \beta \therefore \angle BAC = 2\alpha$$

$$\text{and } \angle BAE = 3\alpha \text{ In } \triangle BAC \tan \alpha = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{1}{2} \right)$$

(ii) (c)

Solution:

$$\text{We know that } \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2 \times \frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{\frac{3}{4}} = \frac{4}{3} \therefore 2\alpha = \tan^{-1} \left(\frac{4}{3} \right)$$

(iii) (d)

Solution:

$$\text{Also } \tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha} = \frac{3 \times \frac{1}{2} - \left(\frac{1}{2}\right)^3}{1 - 3\left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{3}{2} - \frac{1}{8}}{1 - \frac{3}{4}} = \frac{\frac{11}{8}}{\frac{1}{4}} = \frac{11}{2} \Rightarrow 3\alpha = \tan^{-1} \left(\frac{11}{2} \right)$$

(iv) (b)

Solution:

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{\frac{1}{2} - \frac{2}{5}}{1 + \frac{1}{2} \times \frac{2}{5}}$$

$$= \frac{1}{12} \left\{ \because \tan \beta = \frac{BC}{AB} = \frac{10}{25} \right\}$$

(v) (c)

Solution:

$$R, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Answer Key-

Multiple Choice questions-

1. Answer: (b) $\frac{\pi}{3}$
2. Answer: (b) $-\frac{\pi}{3}$
3. Answer: (b) $-\frac{5\pi}{6}$
4. Answer: (d) 1
5. Answer: (b) $-\frac{\pi}{2}$
6. Answer: (d) $\frac{x}{\sqrt{1+x^2}}$
7. Answer: (c) 0
8. Answer: (c) $\frac{\pi}{4}$
9. Answer: (d) $-\frac{\pi}{10}$
10. Answer: (a) $\frac{2\pi}{9}$

Very Short Answer:

1. Solution:

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Hence, the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{\pi}{6}$

2. Solution:

$$\begin{aligned} & \cos^{-1}\left(\cos \frac{2\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right) \\ &= \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3} + \frac{\pi}{3} = \pi. \end{aligned}$$

3. Solution:

$$\begin{aligned} \tan^{-1}(\sqrt{3}) &= \frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \sec^{-1}(-2) &= \frac{2\pi}{3} \in \left[0, \frac{\pi}{2}\right] - \left\{\frac{\pi}{2}\right\}. \\ \therefore \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) \\ &= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}. \end{aligned}$$

4. Solution:

$$\begin{aligned} & \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] \\ &= \tan^{-1}\left[2\cos\left(2 \cdot \frac{\pi}{6}\right)\right] \\ &= \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left(2 \cdot \frac{1}{2}\right) \\ &= \tan^{-1}(1) = \frac{\pi}{4}. \end{aligned}$$

5. Solution:

$$\begin{aligned} & \tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}) \\ &= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2} \end{aligned}$$

6. Solution:

$$\sin^{-1} \left(\frac{1}{3} \right) + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow x = 1/3$$

$$[\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}]$$

7. Solution:

$$\sec^{-1} (2) + \operatorname{cosec}^{-1} (y) = \frac{\pi}{2}$$

$$\Rightarrow y = 2 \left[\because \sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2} \right]$$

8. Solution:

$$\sin \left[\frac{\pi}{3} - \sin^{-1} \left(\frac{-1}{2} \right) \right]$$

$$= \sin \left[\frac{\pi}{3} + \sin^{-1} \left(\frac{1}{2} \right) \right]$$

$$[\because \sin^{-1} (-x) = -\sin^{-1} x]$$

$$= \sin \left(\frac{\pi}{3} + \frac{\pi}{6} \right) = \sin \frac{\pi}{2} = 1$$

9. Solution:

$$\mathbf{LHS} = \cos \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2} \right)$$

$$= \cos \left(\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{3}{\sqrt{13}} \right)$$

$$= \cos \left[\cos^{-1} \left(\frac{4}{5} \cdot \frac{3}{\sqrt{13}} - \sqrt{1 - \frac{16}{25}} \sqrt{1 - \frac{9}{13}} \right) \right]$$

$$= \frac{12}{5\sqrt{13}} - \frac{3}{5} \cdot \frac{2}{\sqrt{13}} = \frac{6}{5\sqrt{13}} = \mathbf{RHS.}$$

10. Solution:

$$\text{We have } \tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{x+y}{1-xy} = \frac{\pi}{4} \Rightarrow \frac{x+y}{1-xy} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{x+y}{1-xy} = 1 \Rightarrow x+y = 1-xy.$$

Hence $x + y + xy = 1$.

Short Answer:

1. Solution:

$$\begin{aligned}
 & \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) \text{ if } -\frac{\pi}{4} < x < \frac{\pi}{4} \\
 & = \sin^{-1}\left(\sin x \cdot \frac{1}{\sqrt{2}} + \cos x \cdot \frac{1}{\sqrt{2}}\right) \text{ if } -\frac{\pi}{4} < x < \frac{\pi}{4} \\
 & = \sin^{-1}\left(\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}\right) \text{ if } \\
 & \quad -\frac{\pi}{4} + \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4} \\
 & = \sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right) \text{ if } 0 < \left(x + \frac{\pi}{4}\right) < \frac{\pi}{2} \\
 & \quad \text{i.e. principal value} \\
 & = x + \frac{\pi}{4}.
 \end{aligned}$$

2. Solution:

Let **LHS** = θ .

$$\text{Then } \sin \theta = \sin\left(\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right)\right)$$

$$\begin{aligned}
 & = \sin\left(\cos^{-1}\left(\frac{12}{13}\right)\right) \cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right) \\
 & \quad + \cos\left(\cos^{-1}\left(\frac{12}{13}\right)\right) \sin\left(\sin^{-1}\left(\frac{3}{5}\right)\right)
 \end{aligned}$$

$$= \sqrt{1 - \frac{144}{169}} \sqrt{1 - \frac{9}{25}} + \frac{12}{13} \times \frac{3}{5}$$

$$= \frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5} = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

$$\Rightarrow \theta = \sin^{-1} \frac{56}{65}.$$

$$\text{Hence, } \cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}.$$

3. Solution:

$$\text{L.H.S} = \sin^{-1} \frac{8}{17} + \cos^{-1} \frac{4}{5}$$

$$= \tan^{-1} \frac{8}{15} + \tan^{-1} \frac{3}{4}$$

$$= \tan^{-1} \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \times \frac{3}{4}} = \tan^{-1} \frac{77}{36} = \cot^{-1} \left(\frac{36}{77} \right)$$

= R.H.S

4. Solution:

$$\tan^{-1} \left(\frac{x+1}{x-1} \right) + \tan^{-1} \left(\frac{x-1}{x} \right) = \tan^{-1}(-7)$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1} \times \frac{x-1}{x}} \right] = \tan^{-1}(-7)$$

$$\Rightarrow \frac{x(x+1) + (x-1)^2}{x(x-1) - (x^2-1)} = -7$$

$$\Rightarrow 2x^2 - 8x + 8 = 0$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

$$\Rightarrow (x-2)^2 = 0.$$

Hence, $x = 2$.

5. Solution:

$$2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x)$$

$$\Rightarrow \tan^{-1} \frac{2 \sin x}{1 - \sin^2 x} = \tan^{-1}(2 \sec x)$$

$$\Rightarrow \tan^{-1} \left(\frac{2 \sin x}{\cos^2 x} \right) = \tan^{-1}(2 \sec x)$$

$$\Rightarrow \tan^{-1}(2 \sec x \tan x) = \tan^{-1}(2 \sec x)$$

$$\Rightarrow 2 \sec x \tan x = 2 \sec x$$

$$\tan x = 1 [\because \sec x \neq 0]$$

$$\text{Hence, } x = \frac{\pi}{2}$$

6. Solution:

We have:

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4}\right)$$

$$\cos(\tan^{-1} x) = \sin\left(\sin^{-1} \frac{4}{5}\right)$$

$$\cos(\tan^{-1} x) = \frac{4}{5}$$

$$\tan^{-1} x = \cos^{-1} \frac{4}{5}$$

$$\Rightarrow \tan^{-1} x = \tan^{-1} \frac{3}{4}$$

$$\text{Hence } x = \frac{3}{4}$$

7. Solution:

Put $x = \cos \theta$ in RHS

As $1/2 \leq x \leq 1$

$$\text{RHS} = \cos^{-1}(4\cos^3 \theta - 3\cos \theta),$$

$$= \cos^{-1}(\cos 3\theta) = 3\theta = 3\cos^{-1} x = \text{L.H.S}$$

Long Answer:

1. Solution:

$$\text{LHS} = \cos^{-1} x + \cos^{-1} \left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2} \right]$$

$$= \cos^{-1} x + \cos^{-1} \left[\frac{1}{2} \cdot x + \frac{\sqrt{3}}{2} \cdot \sqrt{1-x^2} \right]$$

$$= \theta + \cos^{-1} \left[\cos \frac{\pi}{3} \cdot \cos \theta + \sin \frac{\pi}{3} \cdot \sin \theta \right]$$

[Putting $x = \cos \theta$ so that $\sqrt{1-x^2} = \sin \theta$]

$$= \theta + \cos^{-1} \left[\cos \left(\frac{\pi}{3} - \theta \right) \right]$$

$$= \theta + \left(\frac{\pi}{3} - \theta \right) = \frac{\pi}{3} = \text{RHS.}$$

2. Solution:

$$\tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right)$$

$$= \tan^{-1} \left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y} \cdot \frac{x-y}{x+y}} \right]$$

$$\left[\because \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right) \right]$$

$$= \tan^{-1} \left[\frac{x(x+y) - y(x-y)}{y(x+y) + x(x-y)} \right]$$

$$= \tan^{-1} \left[\frac{x^2 + xy - xy + y^2}{xy + y^2 + x^2 - xy} \right]$$

$$= \tan^{-1} \left(\frac{x^2 + y^2}{x^2 + y^2} \right) = \tan^{-1} (1) = \frac{\pi}{4}.$$

3. Solution:

$$\begin{aligned}
 \text{LHS} &= \left(\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) \right) + \tan^{-1}\left(\frac{1}{8}\right) \\
 &= \tan^{-1} \frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{2} \times \frac{1}{5}} + \tan^{-1}\left(\frac{1}{8}\right) \\
 &= \tan^{-1} \frac{5+2}{10-1} + \tan^{-1}\left(\frac{1}{8}\right) \\
 &= \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1} \frac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{9} \times \frac{1}{8}} \\
 &= \tan^{-1} \frac{56+9}{72-7} = \tan^{-1}\left(\frac{65}{65}\right) = \tan^{-1}(1) = \frac{\pi}{4} \\
 &= \text{RHS.}
 \end{aligned}$$

4. Solution:

$$\begin{aligned}
 2 \sin^{-1}\left(\frac{3}{5}\right) &= \sin^{-1}\left(2 \cdot \frac{3}{5} \sqrt{1 - \frac{9}{25}}\right) \\
 & \quad [\because 2 \sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})] \\
 &= \sin^{-1}\left(2 \times \frac{3}{5} \times \frac{4}{5}\right) = \sin^{-1}\left(\frac{24}{25}\right) \quad \dots(1) \\
 &= \tan^{-1}\left(\frac{24}{7}\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, LHS} &= 2 \sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) \\
 &= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right) \quad [\text{Using (1)}] \\
 &= \tan^{-1} \frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \times \frac{17}{31}} = \tan^{-1} \frac{744 - 119}{217 + 408} \\
 &= \tan^{-1} \frac{625}{625} = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS.}
 \end{aligned}$$

Assertion and Reason Answers:

1. (d) A is false and R is true.

Solution :

Given $R = \{(1,1), (1,3), (3,1), (3,3), (3,5)\}$

We know that Relation 'R' is reflexive on set A if $\forall a \in A, (a,a) \in R$

Here set $A = \{1,3,5\}$

$(1,1) \in R, (3,3) \in R$

but $(5,5) \notin R$

$\therefore R$ is not reflexive

\therefore Assertion A is false By definition of transition Relation, It is clear that given Reason R is true. Hence option (d) is the correct answer.

2. (c) A is true but R is false.

Solution:

Given $R = \{(a,b) : |a-b| < 2\}$

$= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4)\}$.

Here $(1,1) \in R, (2,2) \in R, (3,3) \in R, (4,4) \in R, (5,5) \in R$

\Rightarrow Relation R is reflexive on set $A = \{1,2,3,4,5\}$

\therefore Assertion A is true We know that relation R is reflexive if $(a,a) \in R \forall a \in A$

\therefore Given Reason R is false Hence option (c) is the correct answer.