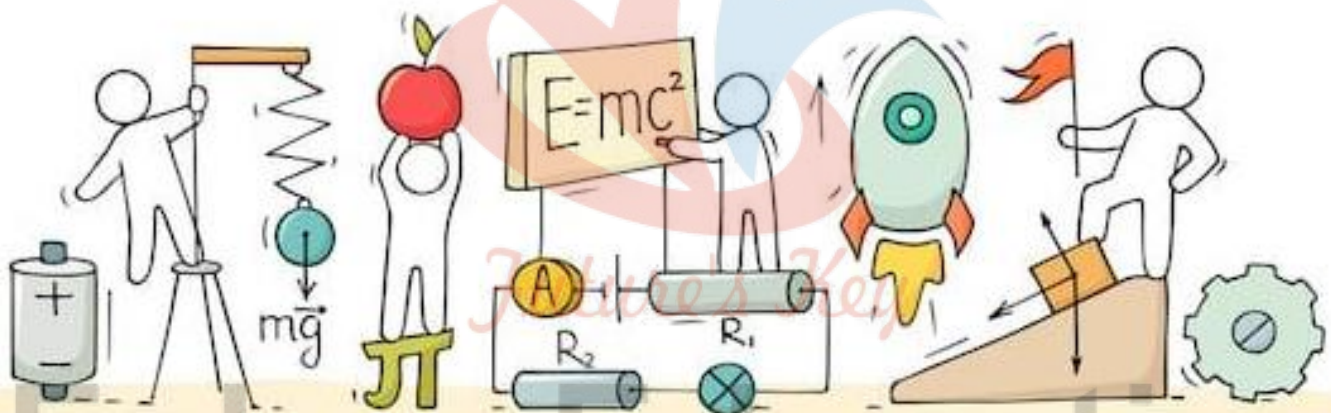


PHYSICS

Chapter 12: Kinetic Theory



Fukey Education

Kinetic Theory

Introduction

What is Kinetic Theory

Kinetic theory explains the behaviour of gases based on the idea that the gas consists of rapidly moving atoms or molecules.

In solids the molecules are very tightly packed as inter molecular space is not present. In liquids inter molecular spaces are more as compared to solids and in gases the molecules are very loosely packed as intermolecular spaces are very large.

The random movement of molecules in a gas is explained by kinetic theory of gases.

We will also see that why kinetic theory is accepted as a successful theory.

Kinetic theory explains the following:

Molecular interpretation of pressure and temperature can be explained.

It is consistent with gas laws and Avogadro's hypothesis.

Correctly explains specific heat capacities of many gases.



Assumptions of Kinetic Theory of Gases

All gas molecules constantly move in random directions.

The size of molecules is very less than the separation between the molecules.

The molecules of the sample do not exert any force on the walls of the container during the collision when the gas sample is contained.

It has a very small time interval of collision between two molecules, and between a molecule and the wall.

Collisions between molecules and wall and even between molecules are elastic in nature.

Newton's laws of motion can be seen in all the molecules in a certain gas sample.

With due course of time, a gas sample comes to a steady state. The molecule's distribution and the density of molecules do not depend on the position, distance and time.

Kinetic Theory of Gases

The kinetic theory of gases relates the macroscopic property of the gas, like – Temperature, Pressure, and Volume to the microscopic property of the gas, like – speed, momentum, and position. In this model, the atoms and molecules are continually in random motion, constantly colliding with one another and the walls of the container within which the gas is enclosed. It is this motion that results in physical properties such as heat and pressure. In this article, let us delve deeper into the kinetic theory of gases.

Molecular nature of matter

John Dalton

Atomic hypothesis was given by many scientists. According to which everything in this universe is made up of atoms.

Atoms are little particles that move around in a perpetual order attracting each other when they are little distance apart.

But if they are forced very close to each other then they rebel.

For example: - Consider a block of gold. It consists of molecules which are constantly moving.

Dalton's atomic theory is also referred as the molecular theory of matter. This theory proves that matter is made up of molecules which in turn are made up of atoms.

According to Gay Lussac's law when gases combine chemically to yield another gas, their volumes are in ratios of small integers.

Avogadro's law states that the equal volumes of all gases at equal temperature and pressure have the same number of molecules.

Conclusion: - All these laws proved the molecular nature of gases.

Dalton's molecular theory forms the basis of Kinetic theory.

Why was Dalton's theory a success?

Matter is made up of molecules, which in turn are made up of atoms.

Atomic structure can be viewed by an electron microscope.

Solids, Liquids, Gases in terms of molecular structure

Basis of Difference	Solids	Liquids	Gases
Inter Atomic Distance (distance between molecules).	Molecules are very tightly packed. Inter atomic distance is minimum.	Molecules are not so tightly packed. Inter atomic distance is more as compared to	Molecules are loosely packed. Free to move. Inter atomic distance is maximum.

		solids.	
Mean Free Path is the average distance a molecule can travel without colliding.	No mean free path.	Less mean free path.	There is mean free path followed by the molecules.

Behaviour of Gas Molecules

The behaviour of gas molecules is dependent on the properties and laws obeyed by the molecules of the gas. The distribution of molecules in a gas is very different from the distribution of molecules in liquids and solids. There are five properties and five gas laws that govern the behaviour of gas molecules.

Gas is defined as a homogeneous fluid which has low density and low viscosity and the volume of the gas is assumed to have the volume equal to the volume of the vessel. The classification of gases are:

Ideal gas

Non-ideal gas or real gas

Following are the properties of gases:

Property	Symbol	Common units
Density	d	g.l^{-1}
Temperature	T	K
Pressure	P	mm Hg
Volume	V	cm
Amount of gas	n	mol

Kinetic Theory of Gases

The behaviour of gas molecules is explained with the help of the kinetic theory of gases. It is the study of gas molecules at the macroscopic level. Following are the five postulates of the kinetic theory of gases:

Gas is the composition of a large number of molecules that are constantly in a random movement.

The volume of the molecules is negligible as the distance between the gas molecules is greater than the size of the molecules.

The intermolecular interactions are also negligible.

The collision of molecules with each other and with the walls of the container is always elastic.

The average kinetic energy of all the molecules is dependent on the temperature.

Boyle's Law

According to Boyle's law, the volume of the gas is inversely related to pressure when the amount of gas is fixed at a constant temperature.

$$P \propto \frac{1}{V}$$

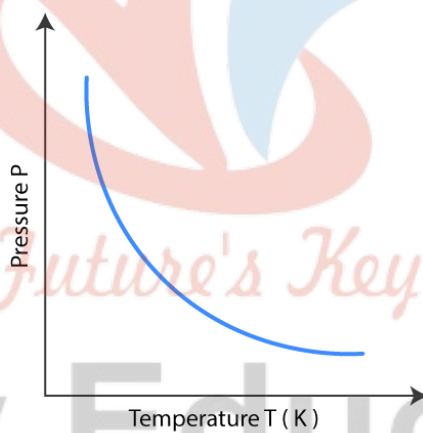
$$PV = \text{constant}$$

$$P_1V_1 = P_2V_2 = P_3V_3 = \text{constant}$$

Where,

P is the pressure of a gas.

V is the volume of gas.



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Charles's Law

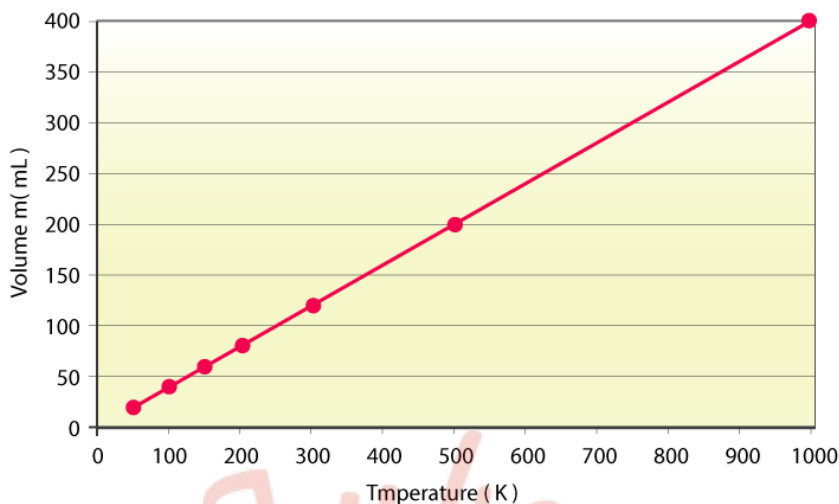
According to Charles's law, the volume of the gas with a fixed mass is directly proportional to the temperature.

$$V \propto T$$

Where,

T is the temperature of a gas.

V is the volume of gas.



CHARLES'S LAW

Gay-Lussac's Law

According to Gay-Lussac's law, when the volume of the gas is constant, the pressure of a given mass of gas varies directly with the absolute temperature of the gas.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

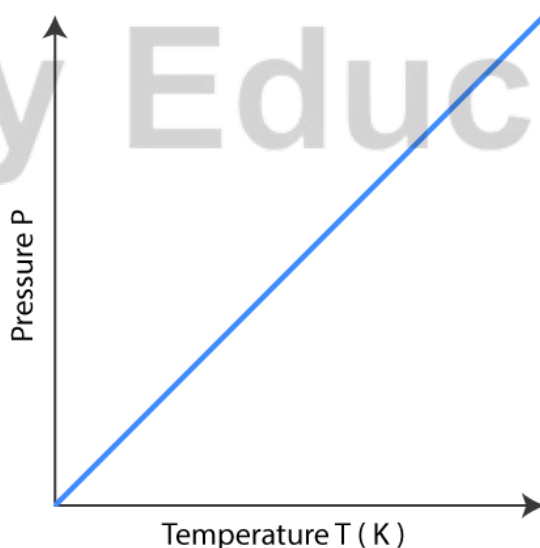
Where,

T_1 is the initial temperature.

P_1 is the initial pressure.

T_2 is the final temperature.

P_2 is the final pressure.



GAY-LUSSAC'S LAW

Avogadro's Law

According to Avogadro's law, when the pressure and temperature of the given gas are constant, then the number of moles and the volume of the gas are in a direct relationship.

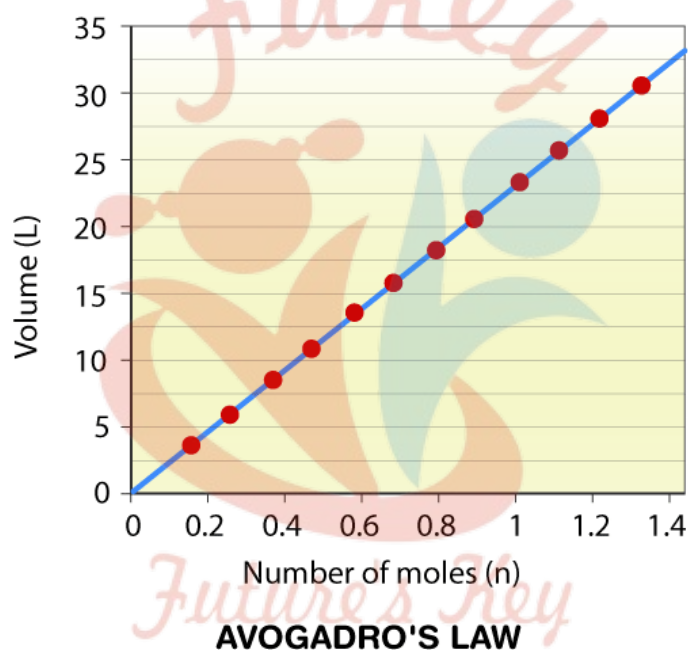
$$V \propto n \text{ or } \frac{V}{n} = k$$

Where,

V is the volume of the gas.

n is the number of moles.

k is the proportionality constant.



Ideal Gas Law

According to ideal gas law, the product of pressure and volume of one gram molecule of an ideal gas is equal to the product of a number of moles of the gas, universal gas constant and the absolute temperature.

$$PV = nRT = NkT$$

Where,

P is the pressure of the gas.

V is the volume of the gas.

n is the number of moles.

R is the universal gas constant = $8.3145 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$

T is the temperature of the gas

N is Avogadro's number, $N_A = 6.0221 \times 10^{23}$

Specific Heat

Specific heat, C_{sp} , is the amount of heat required to change the heat content of exactly 1 gram of a material by exactly 1°C .

Specific heat values can be determined in the following way: When two materials, each initially at a different temperature, are placed in contact with one another, heat always flows from the warmer material into the colder material until both the materials attain the same temperature. From the law of conservation of energy, the heat gained by the initially colder material must equal the heat lost by the initially warmer material.

We know that when heat energy is absorbed by a substance, its temperature increases. If the same quantity of heat is given to equal masses of different substances, it is observed that the rise in temperature for each substance is different. This is due to the fact that different substances have different heat capacities. So heat capacity of a substance is the quantity of the heat required to raise the temperature of the whole substance by one degree. If the mass of the substance is unity then the heat capacity is called Specific heat capacity or the specific heat.

Specific Heat Capacity Formula

$$Q = C m \Delta t$$

Where,

Q = quantity of heat absorbed by a body

m = mass of the body

Δt = Rise in temperature

C = Specific heat capacity of a substance depends on the nature of the material of the substance.

S.I unit of specific heat is $\text{J kg}^{-1} \text{K}^{-1}$.

Specific Heat Capacity Unit

Heat capacity = Specific heat x mass

Its S.I unit is J K^{-1} .

Monatomic Gases

“Monatomic” is a combination of two words “mono”, and “atomic” means a single atom. This term is used in both Physics and Chemistry and is applied to the gases as monatomic gases. In the gaseous phase at sufficiently high temperatures, all the chemical elements are monatomic gases.

Noble gases are monatomic gases as they are unreactive, which is a property of these gases.

They do find applications in daily life like

Helium is used in filling balloons as their density is lower than the air's.

Neon is used for creating advertising signs as they glow when electricity flows through them.

Argon is used in a light bulb to prevent the burning of the filament as it is unreactive

Diatomic Molecules

Diatomic molecules are those molecules that are composed of only two atoms. If a diatomic molecule is composed of the same element, it is known as a homonuclear, and if it is composed of two different elements, it is known as heteronuclear.

Polyatomic Ion

A polyatomic ion is also known as a molecular ion that is composed of two or more covalently bonded atoms. It is also referred to as a radical.

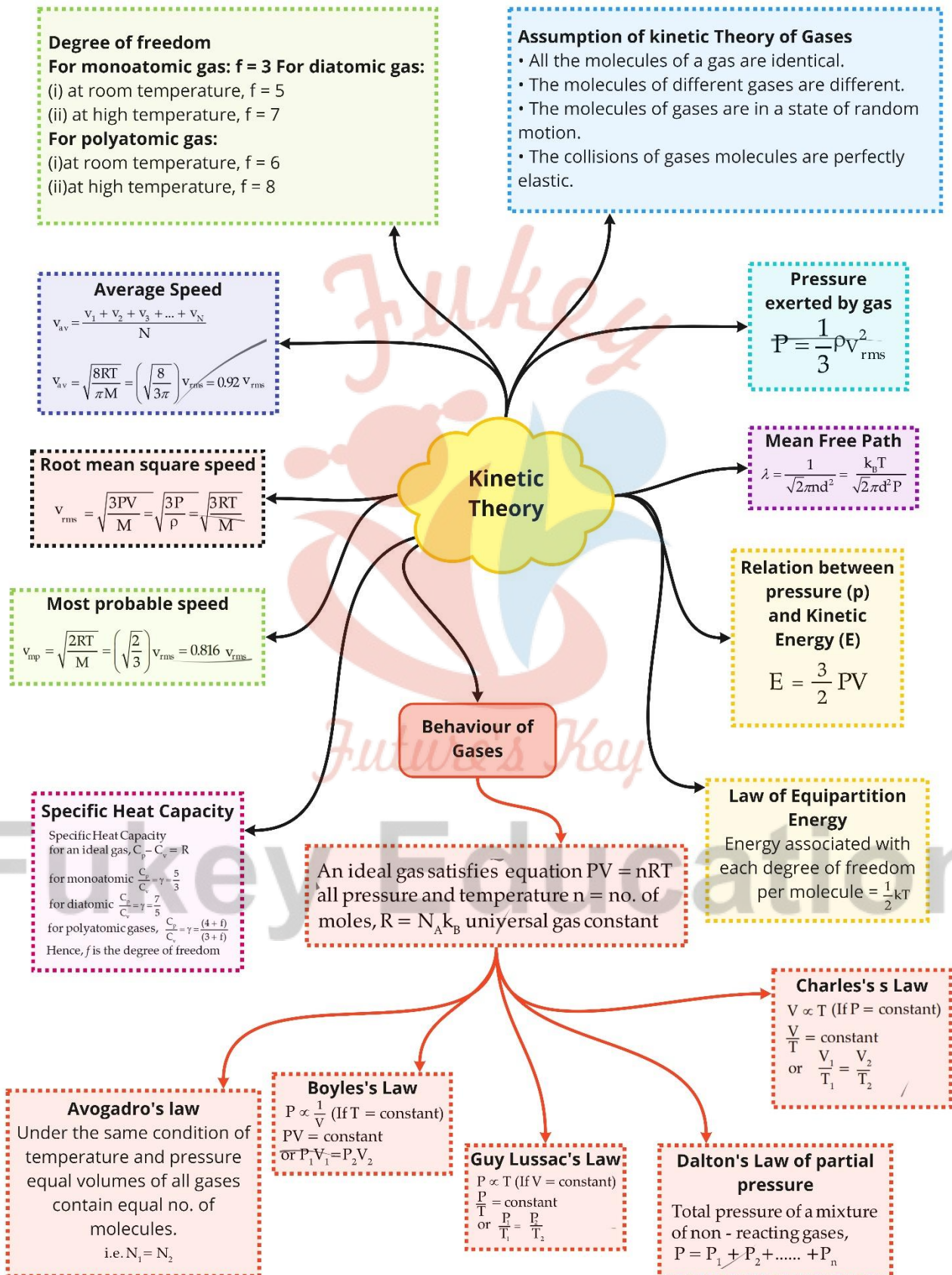
Top Formulae



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Boyle's law	$PV = \text{constant}$
Charles' law	$V/T = \text{constant}$
Gay-Lussac's law	$P/T = \text{constant}$
Gas equation	$PV = \mu RT$, where μ is the number of moles of the given gas.
Pressure exerted by gas	$P = \frac{1}{3}nm\overline{v^2}$
Mean KE of translation per molecule of a gas	$= \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$
Mean KE of translation per mole of a gas	$= \frac{1}{2}M\overline{v^2} = \frac{3}{2}RT = \frac{3}{2}NkT$
Total KE per mole of gas	$= \frac{n}{2}RT$, where n is the number of degrees of freedom of each molecule.
rms speed	$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}}$
Effect of temperature	$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$
Mean free path	$\bar{l} = \frac{k_B T}{\sqrt{2} d p} = \frac{1}{\sqrt{2} \pi d^2 n}$ where n is the number of molecules per unit volume of the gas.
Collision frequency	$f = v / \lambda$

Class : 11th Physics
Chapter- 13 : Kinetic Theory



Important Questions

Multiple Choice questions-

Question 1. A room temperature the r.m.s. velocity of the molecules of a certain diatomic gas is found to be 1930 m/sec. the gas is

- (a) H^2
- (b) F^2
- (c) O^2
- (d) Cl^2

Question 2. Energy supplied to convert unit mass of substance from solid to liquid state at its melting point is called

- (a) Latent heat of fusion
- (b) Evaporation
- (c) Solidification
- (d) Latent heat of fission

Question 3. On any planet, the presence of atmosphere implies [n_{rms} = root mean square velocity of molecules and v_e = escape velocity]

- (a) $n_{rms} \ll v_e$
- (b) $n_{rms} > v_e$
- (c) $n_{rms} = v_e$
- (d) $n_{rms} = 0$

Question 4. Calculate the RMS velocity of molecules of a gas of which the ratio of two specific heats is 1.42 and velocity of sound in the gas is 500 m/s

- (a) 727 m/s
- (b) 527 m/s
- (c) 927 m/s
- (d) 750 m/s

Question 5. The r.m.s. speed of the molecules of a gas in a vessel is 200 m/s. if 25% of the gas leaks out of the vessel, at constant temperature, then the r.m.s. speed of the remaining molecules will be

- (a) 400 m/s

- (b) 150 m/s
- (c) 100 m/s
- (d) 200 m/s

Question 6. A gas is taken in a sealed container at 300 K. it is heated at constant volume to a temperature 600 K. the mean K.E. of its molecules is

- (a) Halved
- (b) Doubled
- (c) Tripled
- (d) Quadrupled

Question 7. Moon has no atmosphere because

- (a) It is far away from the surface of the earth
- (b) Its surface temperature is 10°C
- (c) The r.m.s. velocity of all the gas molecules is more than the escape velocity of the moons surface
- (d) The escape velocity of the moons surface is more than the r.m.s velocity of all molecules

Question 8. A unit mass of solid converted to liquid at its melting point. Heat is required for this process is:

- (a) Specific heat
- (b) Latent heat of vaporization
- (c) Latent heat of fusion
- (d) External latent heat

Question 9. One mole of ideal gas required 207 J heat to rise the temperature by 10°K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10°K the heat required is ($R = 8/3 \text{ J/mole } ^{\circ}\text{K}$)

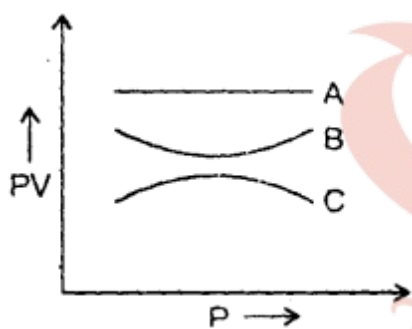
- (a) 1987 J
- (b) 29 J
- (c) 215.3 J
- (d) 124 J

Question 10. The r.m.s velocity of the molecules of an ideal gas is C at a temperature of 100K. at what temperature is r.m.s. velocity will be doubted?

- (a) 200 K
- (b) 400 K
- (c) 300 K
- (d) 50 K

Very Short:

1. What does gas constant R signify? What is its value?
2. What is the nature of the curve obtained when:
 - (a) Pressure versus reciprocal volume is plotted for an ideal gas at a constant temperature.
 - (b) Volume of an ideal gas is plotted against its absolute temperature at constant pressure.
3. The graph shows the variation of the product of PV with the pressure of the constant mass of three gases A, B and C. If all the changes are at a constant temperature, then which of the three gases is an ideal gas? Why?



4. On the basis of Charles's law, what is the minimum possible temperature?
5. What would be the ratio of initial and final pressures if the masses of all the molecules of a gas are halved and their speeds are doubled?
6. Water solidifies into ice at 273 K. What happens to the K.E. of water molecules?
7. Name three gas laws that can be obtained from the gas equation.
8. What is the average velocity of the molecules of a gas in equilibrium?
9. A vessel is filled with a mixture of two different gases. Will the mean kinetic energies per molecule of both gases be equal? Why?
10. The density of a gas is doubled, keeping all other factors unchanged. What will be the effect on the pressure of the gas?

Short Questions:

1. Why cooling is caused by evaporation?
2. On reducing the volume of the gas at a constant temperature, the pressure of the gas increases. Explain on the basis of the kinetic theory of gases.
3. Why temperature less than absolute zero is not possible?
4. There are n molecules of a gas in a container. If the number of molecules is increased to $2n$, what will be:
 - (a) the pressure of the gas.
 - (b) the total energy of the gas.
 - (c) r.m.s. speed of the gas molecules.
5. Equal masses of O_2 and He gases are supplied equal amounts of heat. Which gas will undergo a greater temperature rise and why?
6. Two bodies of specific heats S_1 and S_2 having the same heat capacities are combined to form a single composite body. What is the specific heat of the composite body?
7. Tell the degree of freedom of:
 - (a) Monoatomic gas moles.
 - (b) Diatomic gas moles.
 - (c) Polyatomic gas moles.
8. State law of equipartition of energy.
9. Explain why it is not possible to increase the temperature of gas while keeping its volume and pressure constant?
10. A glass of water is stirred and then allowed to stand until the water stops moving. What has happened to the K.E. of the moving water?

Long Questions:

1. Calculate r.m.s. the velocity of hydrogen at N.T.P. Given the density of hydrogen = 0.09 kg m^{-4} .
2. Calculate the temperature at which r.m.s. the velocity of the gas molecule is double its value at 27°C , the pressure of the gas remaining the same.
3. Calculate the K.E./mole of a gas at N.T.P. Density of gas at N.T.P. = 0.178 g dm^{-3} and molecular weight = 4.
4. Calculate the diameter of a molecule if $n = 2.79 \times 10^{25}$ molecules per m^3 and mean free path = $2.2 \times 10^{-8} \text{ m}$.

5. Calculate the number of molecules in 1 cm^3 of a perfect gas at 27°C and at a pressure of 10 mm of Hg. Mean K.E. of a molecule at $27^\circ\text{C} = 4 \times 10^{25} \text{ J}$. $\rho_{\text{Hg}} = 13.6 \times 10^3 \text{ kg m}^{-3}$.

Assertion Reason Questions:

1. **Directions:** Choose the correct option from the following:

- (a) Both A and R are true, and R is the correct explanation of A
- (b) Both A and R are true, but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false and R is also false

Assertion (A): The number of degrees of freedom of a linear triatomic molecules is 7.

Reason (R): The number of degrees of freedom depends on number of particles in the system.

2. **Directions:** Choose the correct option from the following:

- (a) Both A and R are true, and R is the correct explanation of A
- (b) Both A and R are true, but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false and R is also false

Assertion (A): Absolute zero is not the temperature corresponding to zero energy.

Reason (R): The temperature at which no molecular motion ceases is called absolute zero temperature.

✓ Answer Key:

Multiple Choice Answers-

1. Answer: (a) H^2
2. Answer: (a) Latent heat of fusion
3. Answer (a) $n_{\text{rms}} \ll n_e$
4. Answer: (a) 727 m/s
5. Answer: (d) 200 m/s
6. Answer: (b) Doubled
7. Answer: (c) The r.m.s. velocity of all the gas molecules is more than the escape velocity of the moon's surface
8. Answer: (c) Latent heat of fusion

9. Answer: (d) 124 J
10. Answer: (b) 400 K

Very Short Answers:

- Answer: The universal gas constant (R) signifies the work done by (or on) a gas per mole per kelvin. Its value is $8.31 \text{ J mol}^{-1} \text{ K}$
- Answer: (a) It is a straight line.
(b) It is a straight line.
- Answer: A is an ideal gas because PV is constant at constant temperature for an ideal gas.
- Answer: -273.15°C .
- Answer: 1: 2 ($\because P = \frac{13mn}{V} C^2$)
- Answer: It is partly converted into the binding energy of ice.
- Answer:
 - Boyle's law
 - Charle's law
 - Gay Lussac's law.
- Answer: Zero.
- Yes. This is because the mean K.E. per molecule i.e. $\frac{3}{2} kT$ depends only upon the temperature.
- It will be doubled. ($\because P \propto \rho$ if other factors are constant).

Short Questions Answers:

- Answer: Evaporation occurs on account of faster molecules escaping from the surface of the liquid. The liquid is therefore left with molecules having lower speeds. The decrease in the average speed of molecules results in lowering the temperature and hence cooling is caused.
- Answer: On reducing the volume, the space for the given number of molecules of the gas decreases i.e. no. of molecules per unit volume increases. As a result of which more molecules collide with the walls of the vessel per second and hence a larger momentum is transferred to the walls per second. Due to which the pressure of gas increases.
- Answer: According to the kinetic interpretation of temperature, absolute temperature means the kinetic energy of molecules.

As heat is taken out, the temperature falls and hence velocity decreases. At absolute zero, the velocity of the molecules becomes zero i.e. kinetic energy becomes zero. So no more decrease in K.E. is possible, hence temperature cannot fall further.

4. Answer: (a) We know that

$$P = \frac{1}{3} mnC^2.$$

where n = no. of molecules per unit volume.

Thus when no. of molecules is increased from n to $2n$, no. of molecules per unit volume (n) will increase from n to $2n$

$\frac{n}{V}$ to $\frac{2n}{V}$, hence pressure will become double.

(b) The K.E. of a gas molecule is,

$$\frac{1}{2} mC^2 = \frac{3}{2} kT$$

If the no. of molecules is increased from n to $2n$. There is no effect on the average K.E. of a gas molecule, but the total energy is doubled.

$$\text{r.m.s speed of gas is } C_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3P}{mn}}$$

When n is increased from n to $2n$. both n and P become double and the ratio $\frac{P}{n}$ remains unchanged. So there will be no effect of increasing the number of molecule from n to $2n$ on r.m.s. speed of gas molecule.

5. Answer: Helium is monoatomic while O_2 is diatomic. In the case of helium, the supplied heat has to increase only the translational K.E. of the gas molecules.

On the other hand, in the case of oxygen, the supplied heat has to increase the translations, vibrational and rotational K.E. of gas molecules. Thus helium would undergo a greater temperature rise.

6. Answer: Let m_1 and m_2 be the masses of two bodies having heat capacities S_1 and S_1 respectively.

$$\therefore (m_1 + m_2)S = m_1S_1 + m_2S_2 = m_1S_1 + m_1S_1 = 2m_1S_1$$

$$S = \frac{2m_1S_1}{m_1 + m_2}.$$

$$\text{Also, } m_2S_2 = m_1S_1$$

(\because Heat capacities of two bodies are same.)

Or

$$m_2 = \frac{m_1 S_1}{S_2}$$

$$\therefore S = \frac{2 m_1 S_1}{m_1 + \frac{m_1 S_1}{S_2}} = \frac{2 S_1 S_2}{S_1 + S_2}$$

7. Answer: (a) A monoatomic gas possesses 3 translational degrees of freedom for each molecule.

(b) A diatomic gas molecule has 5 degrees of freedom including 3 translational and 2 rotational degrees of freedom.

(c) The polyatomic gas molecule has 6 degrees of freedom (3 translational and 3 rotational).

8. Answer: It states that in equilibrium, the total energy of the system is divided equally in all possible energy modes with each mode i.e. degree of freedom having an average energy equal to $\frac{1}{2} K_B T$.

9. Answer: It is not possible to increase the temperature of a gas keeping volume and pressure constant can be explained as follows:

According to the Kinetic Theory of gases,

$$P = \frac{1}{3} \rho C^2 = \frac{1}{3} \frac{M}{V} C^2$$

$$= \frac{1}{3} \frac{M}{V} kT$$

($\because C^2 = kT$, when k is a constant)

$$T \propto PV$$

Now as T is directly proportional to the product of P and V . If P and V are constant, then T is also constant.

10. Answer: The K.E. of moving water is dissipated into internal energy. The temperature of water thus increases.

Long Questions Answers:

1. Answer:

Here,

$$\begin{aligned}\rho &= 0.09 \text{ kg m}^{-3} \\ P &= 76 \text{ cm of Hg} \\ &= 76 \times 13.6 \times 980 \text{ dyne cm}^{-2} \\ &= 1.01 \times 10^6 \text{ dyne cm}^{-2} \\ &= 1.01 \times 10^5 \text{ Nm}^{-2} \\ C &= ?\end{aligned}$$

Using the relation,

$$\begin{aligned}P &= \frac{1}{3} \rho C^2, \text{ we get} \\ C &= \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}} \\ &= \sqrt{3.37 \times 10^6} \\ C &= 1.836 \times 10^3 \text{ ms}^{-1} \\ &= 1836 \text{ ms}^{-1}.\end{aligned}$$

2. Answer: Let t be the required temperature = ? and C_t, C_{27} be the r.m.s. velocities of the gas molecules at $t^\circ\text{C}$ and 27°C respectively.

$$\frac{C_t}{C_{27}} = 2 \text{ (given)}$$

Also let M = molecular weight of the gas

$$\text{Now } T = t + 273$$

$$\text{and } T_{27} = 27 + 273 = 300 \text{ K}$$

\therefore Using the relation

$$C = \sqrt{\frac{3RT}{M}}, \text{ we get}$$

$$C_t = \sqrt{\frac{3RT}{M}}$$

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$$\text{and } C_{27} = \sqrt{\frac{3RT_{27}}{M}}$$

$$\therefore \frac{C_1}{C_{27}} = \sqrt{\frac{T}{T_{27}}} = \sqrt{\frac{t+273}{300}}$$

$$\text{or } 2 = \sqrt{\frac{t+273}{300}}$$

$$\text{or } 4 = \frac{t+273}{300}$$

$$\text{or } t = 1200 - 273 = 927^\circ\text{C}.$$

3. Answer: Here, $\rho = 0.178 \text{ g dm}^{-3}$
 $= 0.178 \times 10^{-3} \text{ g cm}^{-3}$ ($\because 1 \text{ dm}^3 = 10^{-3} \text{ cm}^3$)
 $= 178 \times 10^{-6} \text{ g cm}^{-3}$

Volume of 1 mole of gas i.e. 4 g of gas = $\frac{\text{Mass}}{\text{Density}}$

$$\text{or } V = \frac{M}{\rho} = \frac{4}{178 \times 10^{-6}} \text{ cm}^3$$

$T = 273 \text{ K}$ at NTP

$$\therefore R = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 4}{178 \times 10^{-6} \times 273}$$

K.E./mole = ?

We know that K.E./mole = $\frac{3}{2} RT$

$$\begin{aligned} &= \frac{3}{2} \times \frac{76 \times 13.6 \times 980 \times 4}{178 \times 10^{-6} \times 273} \times 273 \\ &= 3.42 \times 10^{10} \text{ erg} \\ &= \frac{3.42 \times 10^{10}}{10^7} \text{ J} = 3.42 \times 10^3 \text{ J}. \end{aligned}$$

4. Answer: Here, $n = 2.79 \times 10^{25} \text{ molecules m}^{-3}$

$$\lambda = 2.2 \times 10^{-8} \text{ m}$$

$$d = ?$$

Using the relation.

$$\lambda = \frac{1}{\sqrt{2}} \frac{1}{\pi n d^2}, \text{ we get}$$

$$d^2 = \frac{1}{\sqrt{2}} \frac{1}{\pi n \lambda}$$

$$= \frac{1}{1.414 \times 3.142 \times 2.79 \times 10^{25} \times 2.2 \times 10^{-8}}$$

$$= 0.03666 \times 10^{-7} \text{ m}^2$$

$$= 0.367 \times 10^{-18} \text{ m}^2$$

$$\therefore d = \sqrt{0.367 \times 10^{-18} \text{ m}^2}$$

$$= 0.606 \times 10^{-9} \text{ m} = 606 \text{ nm.}$$

5. Answer: Here, K.E. per molecule at $27^\circ\text{C} = 4 \times 10^{-11} \text{ J}$

Let μ = number of molecules in 1 cm^3 or 10^{-6} m^3

\therefore Mean K.E. per $\text{cm}^3 = \mu \times 4 \times 10^{-11} \text{ J} \dots (i)$

Now K.E. per gram molecule = $\frac{3}{2} RT$

for a perfect gas, $PV = RT$

\therefore K.E. per gram molecule = $\frac{3}{2} PV$

or

K.E. per cm^3 of gas = $\frac{3}{2} PV$

$P = 10 \text{ mm of Hg} = 10^{-2} \text{ m of Hg}$

$= 10^{-2} \times 13.6 \times 10^3 \times 9.8$

$= 136 \times 9.8 \text{ Nm}^{-2} \text{ V}$

$= 1 \text{ cm}^3$

$= 10^{-6} \text{ m}^3$

\therefore K.E per cm^3 of gas = $\frac{3}{2} \times 136 \times 9.8 \times 10^6$

$= 1.969 \times 10^{-3} \text{ J} \dots (ii)$

\therefore from (i) and (ii) we get

$\mu \times 4 \times 10^{-11} = 1.969 \times 10^{-3}$

or

$$\mu = \frac{1.969 \times 10^{-3}}{4 \times 10^{-11}}$$

= 4.92×10^7 molecules.

Assertion Reason Answer:

1. (b) Both A and R are true, but R is NOT the correct explanation of A
2. (a) Both A and R are true, and R is the correct explanation of A

Case Study Questions-

1. Boyle's law is a gas law which states that the pressure exerted by a gas (of a given mass, kept at a constant temperature) is inversely proportional to the volume occupied by it. In other words, the pressure and volume of a gas are inversely proportional to each other as long as the temperature and the quantity of gas are kept constant. For a gas, the relationship between volume and pressure (at constant mass and temperature) can be expressed mathematically as follows. $P \propto (1/V)$ – Where P is the pressure exerted by the gas and V is the volume occupied by it. This proportionality can be converted into an equation by adding a constant, k. Charles law states that the volume of an ideal gas is directly proportional to the absolute temperature at constant pressure. The law also states that the Kelvin temperature and the volume will be in direct proportion when the pressure exerted on a sample of a dry gas is held constant. Charles law and Boyle's law applied to low density gas only. The total pressure of a mixture of ideal gases is the sum of partial pressures. This is Dalton's law of partial pressures.
 - i. **Boyle's law is obeyed by high as well as low density gases. True or False?**
 - a. True
 - b. False
 - ii. **Charles law is states that volume of an ideal gas is directly proportional to temperature at constant**
 - a. Temperature
 - b. Pressure
 - c. Volume
 - d. None of these
 - iii. **State Daltons law of partial pressures**
 - iv. **State Boyle's law**
 - v. **State Charles law**
2. Pressure of an Ideal Gas: according to kinetic theory of gases pressure is given by $P = \frac{1}{3} nmv^2$ Where, n is number of molecules per unit volume, m is mass and v^2 is mean squared speed. Though we choose the container to be a cube, the shape of the vessel really is immaterial. The average kinetic energy of a molecule is proportional to the

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absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating temperature, a macroscopic measurable parameter of a gas (a thermodynamic variable as it is called) to a molecular quantity, namely the average kinetic energy of a molecule. The two domains are connected by the Boltzmann constant and given by $E = kbT$. Where k_b is Boltzmann constant having value of 1.38×10^{-23} joule per Kelvin. We have seen that in thermal equilibrium at absolute temperature T , for each translational mode of motion, the average energy is $\frac{1}{2} k_b T$. The most elegant principle of classical statistical mechanics (first proved by Maxwell) states that this is so for each mode of energy: translational, rotational and vibrational. That is, in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $\frac{1}{2} k_b T$. This is known as the law of equipartition of energy. Accordingly, each translational and rotational degree of freedom of a molecule contributes $\frac{1}{2} k_b T$ to the energy, while each vibrational frequency contributes $2 \times \frac{1}{2} k_b T = k_b T$, since a vibrational mode has both kinetic and potential energy modes.

- i. **Boltzmann constant has value of**
 - a. 1.38×10^{-23} joule per Kelvin.
 - b. 1.38×10^{-28} joule per Kelvin.
 - c. 1.38×10^{-30} joule per Kelvin.
 - d. None of these
- ii. **SI unit of Boltzmann constant is given by**
 - a. Joules per meter
 - b. Joules per Kelvin
 - c. Joules per Newton
 - d. None of these
- iii. **According to kinetic theory give formula for pressure of idea gas.**
- iv. **According to kinetic theory what is average kinetic energy of molecules of ideal gas?**
- v. **What is law of equipartition of energy?**

Case Study Answer-

1. Answer

- i. (a) True
- ii. (b) Pressure

- iii. The total pressure of a mixture of ideal gases is the sum of partial pressures exerted by all the molecules of gas. This is Dalton's law of partial pressures.
- iv. Boyle's law is a gas law which states that at constant temperature the pressure exerted by a gas is inversely proportional to the volume occupied by it. In other words, the pressure and volume of a gas are inversely proportional to each other as long as the temperature and the quantity of gas are kept constant. For a gas, the $P \propto (1/V)$ Where P is the pressure exerted by the gas and V is the volume occupied by it. This proportionality can be converted into an equation by adding a constant k.
- v. Charles law states that the volume of an ideal gas is directly proportional to the absolute temperature at constant pressure.

2. Answer

- i. (a) 1.38×10^{-23} joule per Kelvin.
- ii. (b) Joules per Kelvin
- iii. According to kinetic theory of gases pressure is given by $P = \frac{1}{3} nmv^2$ Where, n is number of molecules per unit volume, m is mass and v^2 is mean squared speed. Though we choose the container to be a cube, the shape of the vessel really is immaterial.
- iv. **The average kinetic energy** of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas and given by $E = \frac{3}{2} k_b T$.
Where k_b is Boltzmann constant having value of 1.38×10^{-23} joule per Kelvin.
- v. We know that for each translational mode of motion, the average energy is $\frac{1}{2} k_b T$. classical statistical mechanics states that in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $\frac{1}{2} k_b T$. This is known as the law of equipartition of energy. Accordingly, each translational and rotational degree of freedom of a molecule contributes $\frac{1}{2} k_b T$ to the energy, while each vibrational frequency contributes $2 \times \frac{1}{2} k_b T = k_b T$, since a vibrational mode has both kinetic and potential energy modes.