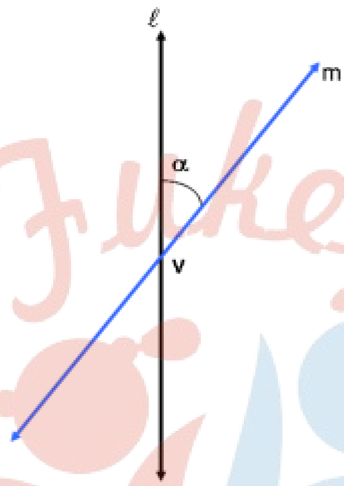
# MATHEMATICS

## Chapter 10: CONIC SECTIONS

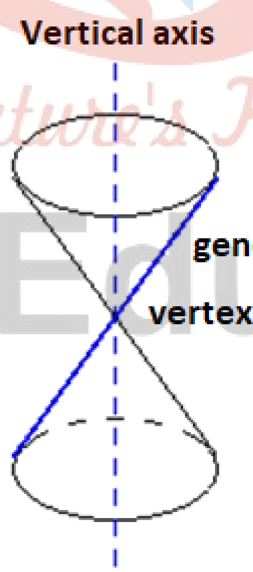
# CONIC SECTIONS

## Key Concepts

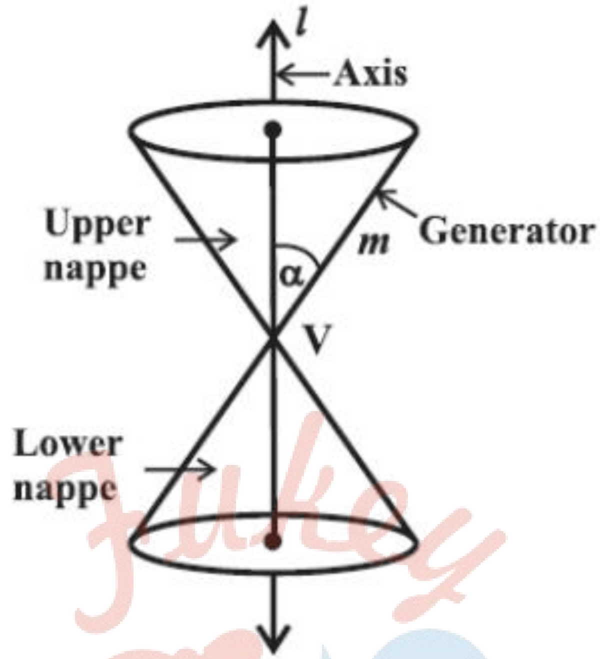
1. Let λ be a fixed vertical line and m be another line intersecting it at a fixed point V and inclined to it atan angle α.




On rotating the line m around the line λ in such a way that the angle α remains constant, the surface generated is a double-napped right-circular hollow cone.




2. The point V is called the vertex; the line λ is the axis of the cone. The rotating line m is called agenerator of the cone. The vertex separates the cone into two parts called nappes.




3. The sections obtained by cutting a double napped cone with a plane are called conic sections. Conicsections are of two types (i) degenerate conics (ii) non-degenerate conics.

4. If the cone is cut at its vertex by the plane, then degenerate conics are obtained

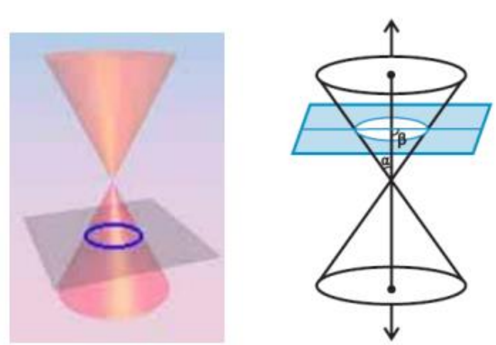
5. If the cone is cut at the nappes by the plane, then non-degenerate conics are obtained.

6. Degenerate conics are points, lines and double lines.

7. Circle, parabola, ellipse and hyperbola are degenerate conics.

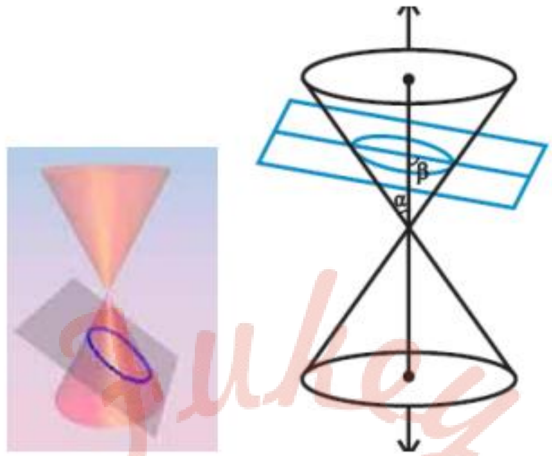
8. When the plane cuts the nappes (other than the vertex) of the cone, degenerate conics are obtained.

(a) When $\beta = 90^\circ$ , the section is a circle.



The plane cuts the cone horizontally.

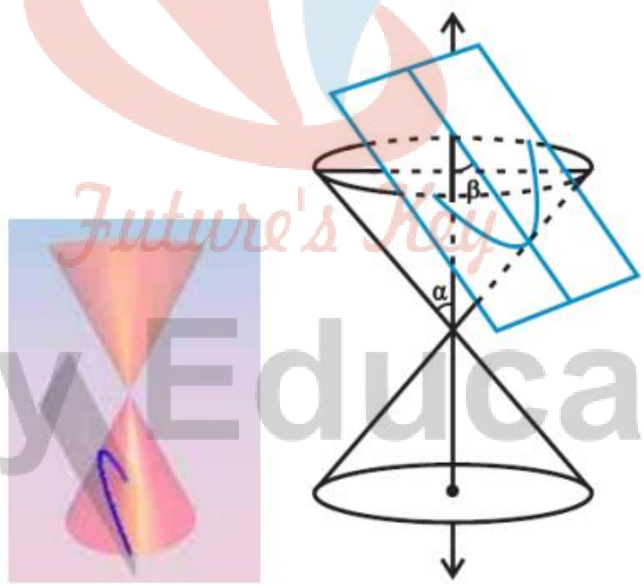
(b) When $\alpha < \beta < 90^{o}$, the section is an ellipse.




**Ellipse**

The plane cuts one part of the cone in an inclined manner.

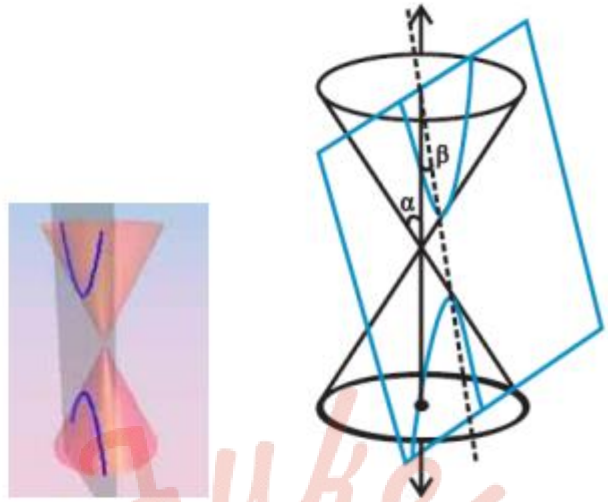
(c) When $\beta = \alpha$, the section is a parabola.




**Parabola**

The plane cuts the cone in such a way that it is parallel to a generator.

(d) When $0 \leq \beta \leq \alpha$, the plane cuts through both the nappes the curve of intersection is a hyperbola.

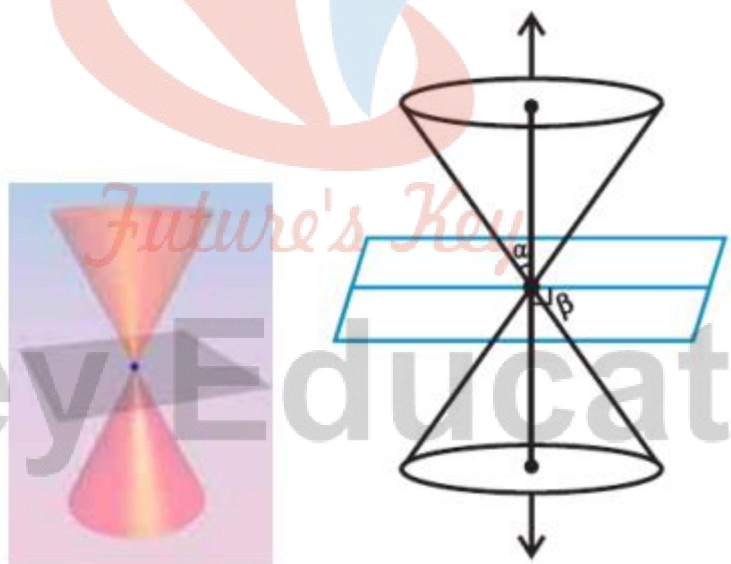


Hyperbola

The plane cuts both parts of the cone.

9. When the plane cuts at the vertex of the cone, we have the following different cases:

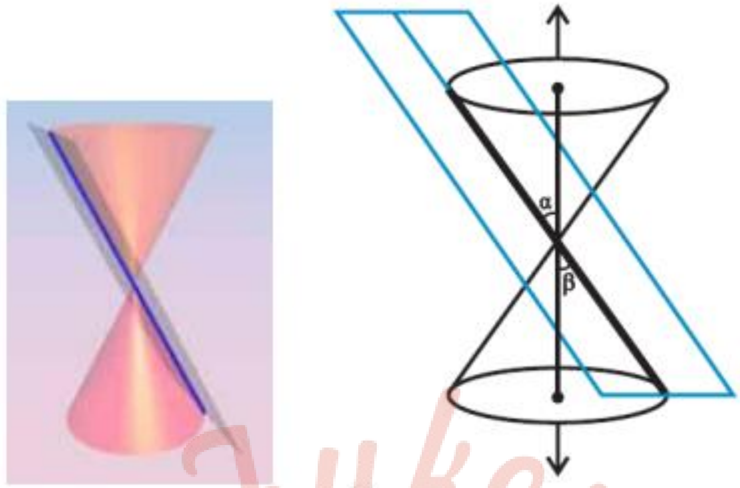
(a) When $\alpha < \beta < 90^o$, the section is a point.



Point

It is a degenerated case of a circle.

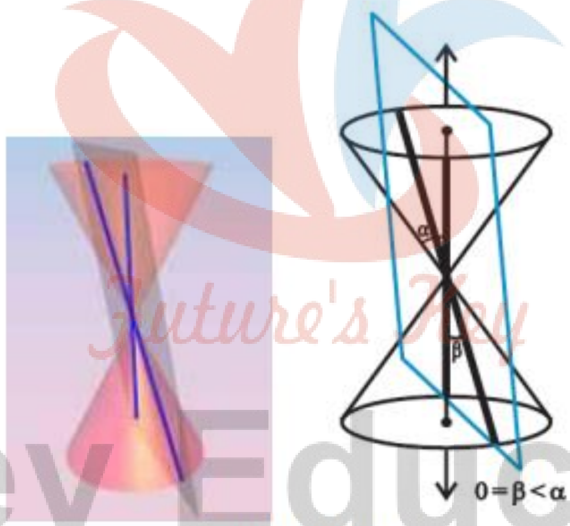
(b) When $\beta = \alpha$ the plane contains a generator of the cone and the section is a straight line.




Line

It is the degenerated case of parabola.

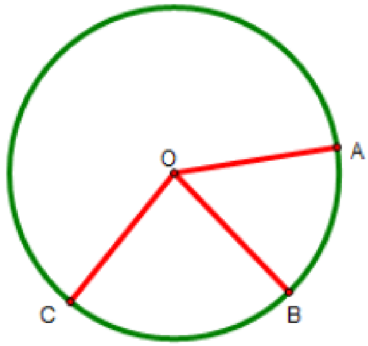
(c) When $0 \le \beta \le \alpha$, the section is a pair of intersecting straight lines. It is the degenerated case of ahyperbola.




Double Line

10. A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

11. The fixed point is called the center of the circle and the distance from the center to a point on thecircle is called the radius of the circle.




In the circle, O is the center and OA = OB = OC are the radii.

12.If the center of a circle is (h, k) and the radius is r, then the equation of the circle is given by $(x - h)^2 + (y - k)^2 = r^2$

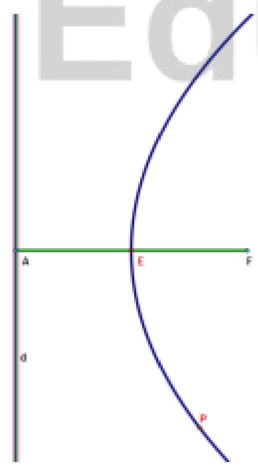
13.A circle with the radius of length zero is a point circle.

14.If the center of a circle is at the origin and radius is r, then the equation of the circle is given by $x^2 + y^2 = r^2$.

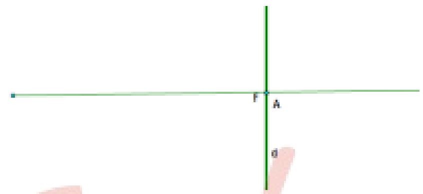
15.If three points lie on the circle and if we prove that the fourth point also lies on the circle, then the fourpoints are concyclic.

16.A **parabola** is the locus of a point, which moves in a plane in such a way that its distance from a fixedpoint (not on the line) in the plane is equal to its distance from a fixed straight line in the same plane.





17. If the fixed point is on the fixed line, then the set of points which are equidistant from the line and focus will be a straight line which passes through the fixed point focus and perpendicular to the givenline. This straight line is the degenerate case of the parabola.

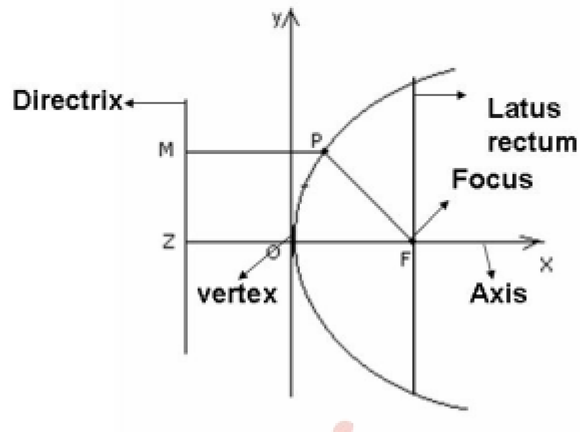



18. The fixed line is called the directrix of the parabola and the fixed point F is called the focus.

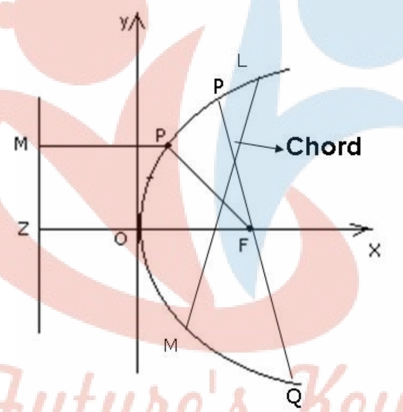
19. 'Para' means 'for' and 'bola' means 'throwing'. The path taken by the trajectory of a rocket artilleryetc. are parabolic.

One of nature's best known approximations to parabolas is the path taken by a body projectedupward and obliquely to the pull of gravity, as in the parabolic trajectory of a golf ball.

20. A line through the focus and perpendicular to the directrix is called the axis of the parabola. The pointof intersection of the parabola with the axis is called the vertex of the parabola.



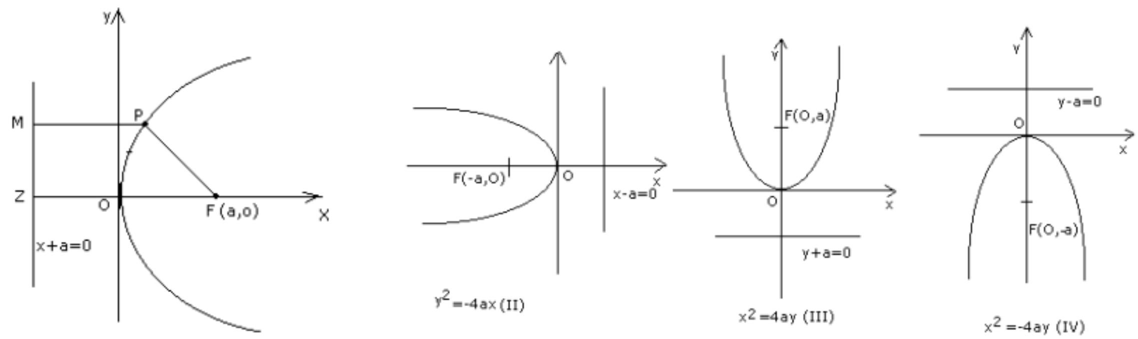

21.A chord of a parabola is the line segment joining any two points on the parabola. If the chord passesthrough the focus, then it is called the focal chord. LM and PQ are both chords but PQ is the focal chord.




22.The chord which passes through the focus is called the focal chord. Focal chord perpendicular to theaxis is called the **latus rectum** of the parabola.

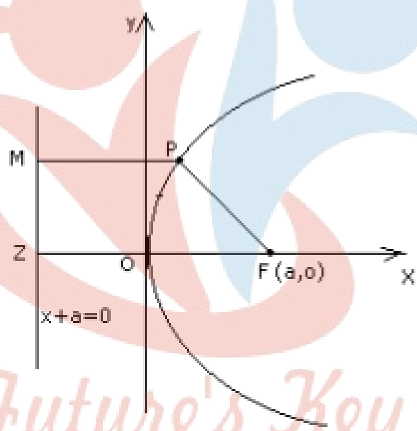
23.Any chord perpendicular to the axis of a parabola is called a double ordinate.

24.The equation of a parabola is simplest if the vertex is at the origin and the axis of symmetry is alongthe X-axis or Y-axis. The four such possible orientations of parabola are shown below.




25. In terms of loci, the conic sections can be defined as follows

Given a line Z and point F not on Z, a conic is the locus of a point P such that the distance from P to F divided by the distance from P to Z is constant, i.e. PF/PM = e, which is a constant called eccentricity. In case of parabola eccentricity, e =1.






26. Parabola is symmetric with respect to its axis. If the equation has a $y^2$ term, then the axis of symmetryis along the X-axis and if the equation has an $x^2$ term, then the axis of symmetry is along the Y-axis.

27. When the axis of symmetry is along the X-axis, the parabola opens to the

(a) Right if the coefficient of x is positive.

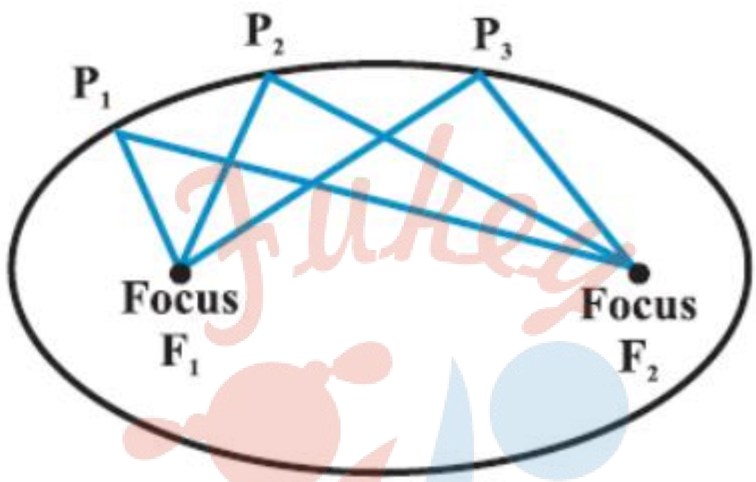
(b) Left if the coefficient of x is negative.

28. When the axis of symmetry is along the Y-axis, the parabola opens

(a) Upwards if the coefficient of y is positive.

(b) Downwards if the coefficient of y is negative.

29. An ellipse is the set of all points in a plane, the sum of whose distance from two fixed points in the plane is constant. These two fixed points are called the foci. For instance, if $F_1$ and $F_2$ are the foci and $P_1$, $P_2$ and $P_3$ are the points on the ellipse then




$P_1F_1 + P_1F_2 = P_2F_1 + P_2F_2 = P_3F_1 + P_3F_2$ is a constant and this constant is more than the distancebetween the two foci.

30. An ellipse is the locus of a point that moves in such a way that its distance from a fixed point (called focus) bears a constant ratio to its distance from a fixed line (called directrix). The ratio e is called theeccentricity of the ellipse. For an ellipse $e<1$.

31. Eccentricity is a measure of the flatness of the ellipse. The eccentricity of a conic section is ameasure of how far it deviates from being circular.

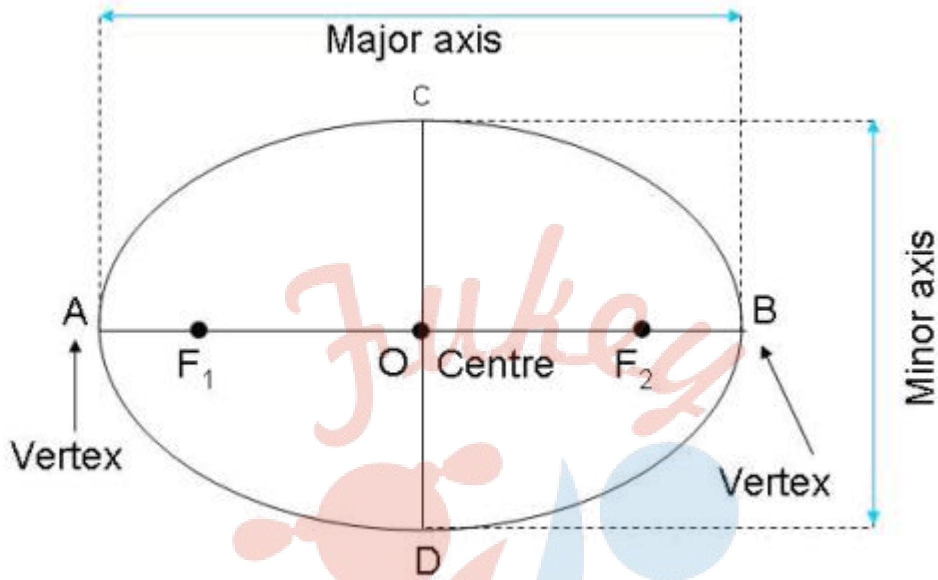
32. Terms associated with ellipse

(a) The midpoint of the line segment joining the foci is called the centre of the ellipse. In the figure, O is the Centre of ellipse. For the simplest ellipse, the centre is at the origin.

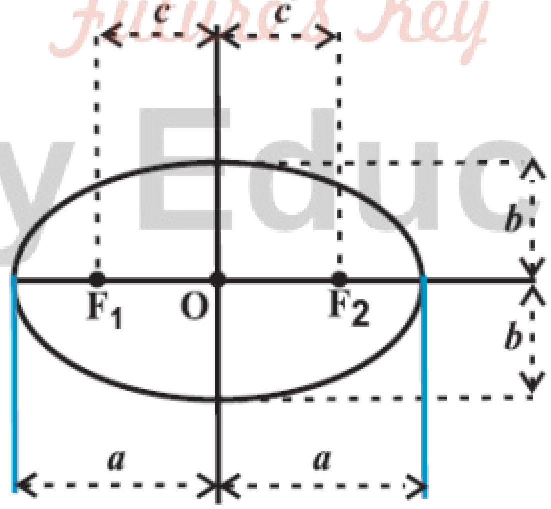
(b) The line segment through the foci of the ellipse is called the major axis and the line segment through the centre and perpendicular to the major axis is called the minor axis. In the figure below AB and in case of the simplest ellipse, the two axes are along the coordinate axes. The two axes intersect at the centre of ellipse.

(c) Major axes represent the longer sections of parabola and the foci lies on major axes.

(d) The end points of the major axis are called the vertices of the ellipse.



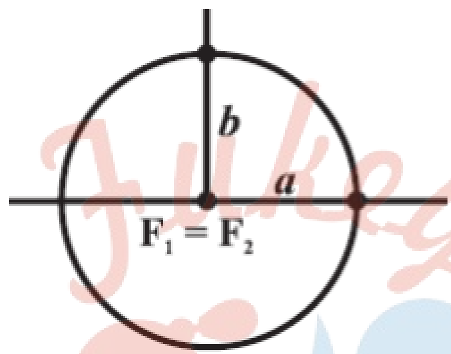

33. If the distance from each vertex on the major axis to the centre be a, then the length of the major axis is 2a. Similarly, if the distance of each vertex on minor axis to the centre is b, the length of the minor axis is 2b. Finally, the distance from each focus to the centre is c. So, the distance between foci is 2c.



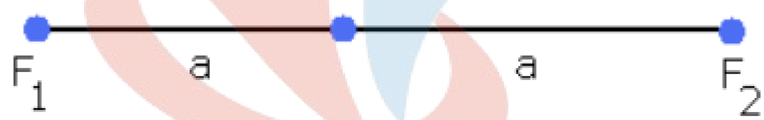

34. Semi-major axis a, semi-minor axis b and the distance of focus from the centre c are connected bythe relation $a^2 = b^2 + c^2$ or $c^2 = a^2 - b^2$.

35. In the equation $c^2 = a^2 - b^2$ if a is fixed and c varies from 0 to a, then the resulting ellipses will vary in shape.

Case (i) When c = 0, both foci merge together with the centre of the ellipse and $a^2 = b^2$, i.e., a = b, andthus the ellipse becomes circle. Thus, the circle is a special case of an ellipse.

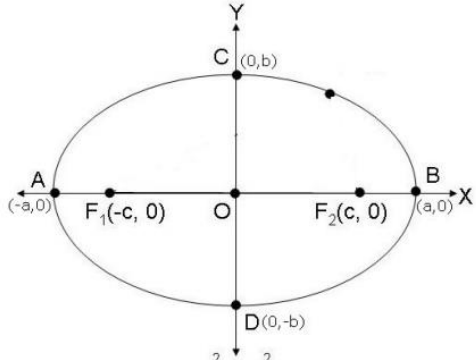



Case (ii) When c = a, b = 0. The ellipse reduces to the line segment $F_1F_2$ joining the two foci.

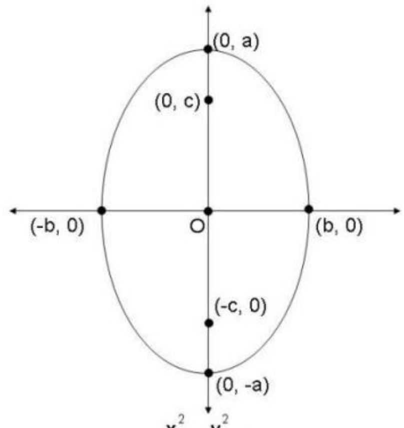



36. The eccentricity of an ellipse is the ratio of the distances from the centre of the ellipse to one of thefoci and one of the vertices of the ellipse. Eccentricity is denoted by e, i.e. $e = \frac{c}{a}$.

37. The standard form of ellipses having the centre at the origin and the major and minor axis ascoordinate axes. There are two possible orientations:




$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$




$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

38. Ellipse is symmetric with respect to both the coordinate axes and across the origin. If (x, y) is a pointon the ellipse, then (−x, y), (x, −y) and (−x, y) are also points on the ellipse.

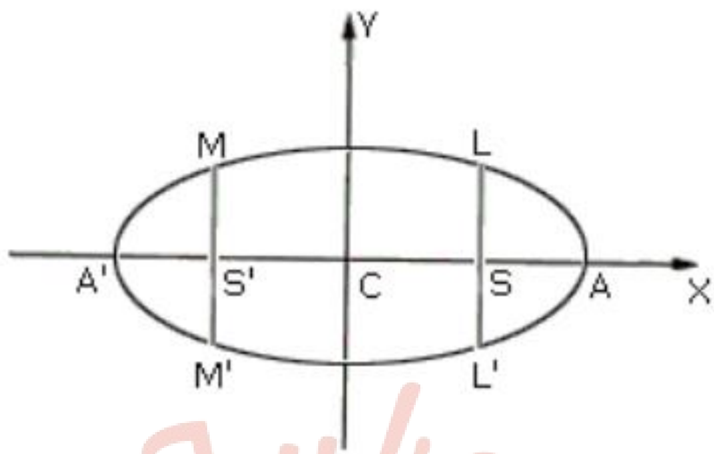
39. Because the ellipse is symmetric across the Y-axis, it follows another point $F_2(-c,0)$ which may be considered as a focus, corresponding to another directrix. Thus, every ellipse has two foci and twodirectrices

40. The distances AA' = 2a and BB' = 2b are called the major and minor axes.

41. Let P be a point on the ellipse and let PN be perpendicular to the major axis AA' such that PN produced meets the ellipse at P'. Thus, PN is called the ordinate of P and PNP' is called the doubleordinate of P.

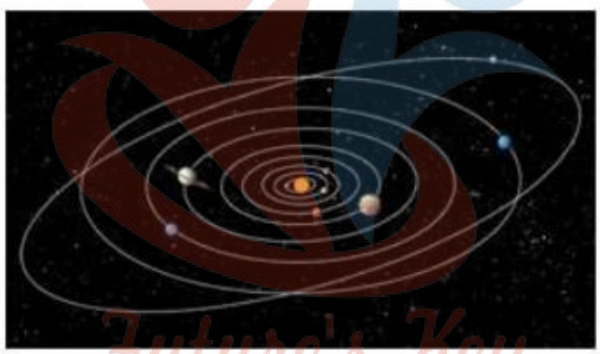
42. The foci always lie on the major axis. The major axis can be determined by finding the intercepts onthe axes of symmetry, i.e. the major axis is along the X-axis if the coefficient of $x^2$ has a larger denominator and it is along the Y-axis if the coefficient of $y^2$ has a larger denominator.

43. Lines perpendicular to the major axis A'A through the foci F1 and F2, respectively, are called latusrectum, i.e. LL’ and MM’ are the latus rectum.




44.The sum of focal distances of any point on an ellipse is a constant and is equal to the major axis.

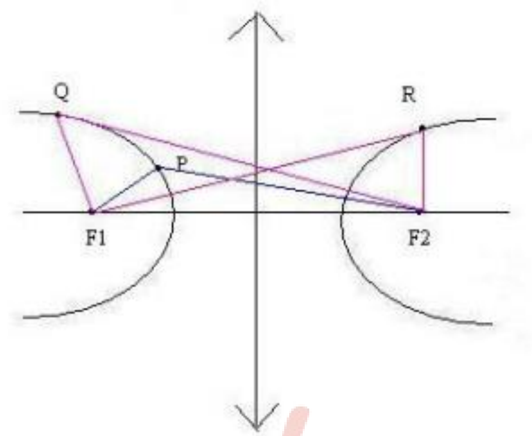
45.A conic ellipse can be seen in the physical world. The orbital of planets is elliptical.



Apart from this, one can see an ellipse at many places since every circle, viewed obliquely, appearselliptical.

If a glass of water is seen from the top or if it is held straight, it appears to be circular but if it is tilted itwill be elliptical.

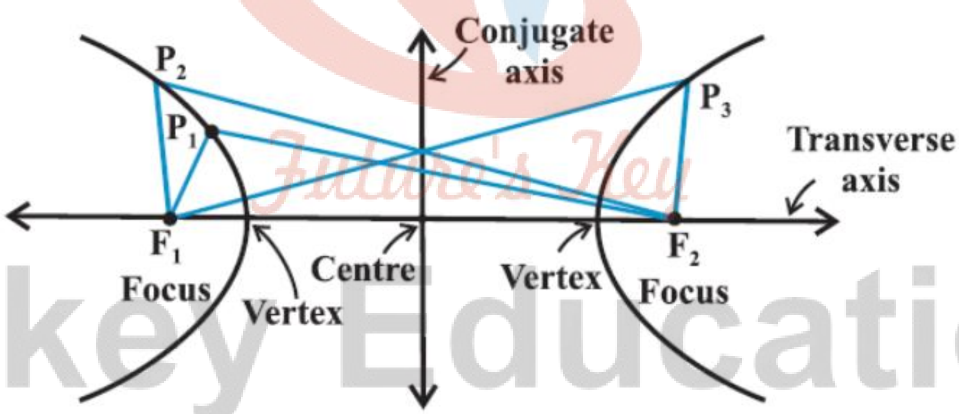
46.A hyperbola is a set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant. The two fixed points are called the foci of the hyperbola.




(Distance to $F_1$) − (distance to $F_2$) = constant

47. A hyperbola is the locus of a point in the plane which moves in such a way that its distance from a fixed point in the plane bears a constant ratio, e > 1, to its distance from a fixed line in the plane. Thefixed point is called **focus**, the fixed line is called **directrix** and the constant ratio e is called the **eccentricity** of the hyperbola.

48. Terms associated with hyperbola




$$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2$$

(a) The midpoint of the line segment joining the foci is called the centre of the hyperbola.

(b) The line through the foci is called the transverse axis and the line through the centre and perpendicular to the transverse axis is called the conjugate axis.

(c) The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.

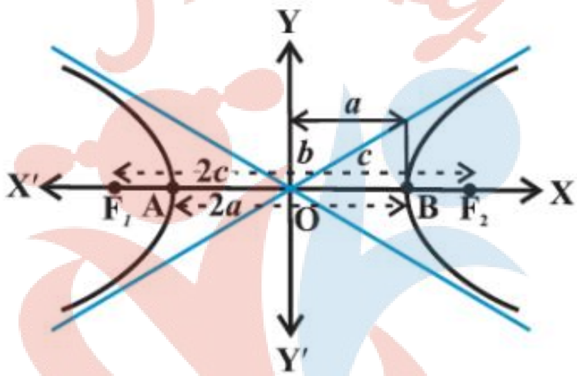
49. The hyperbola is perfectly symmetrical about the centre O.

50. Let the distance of each focus from the centre be c, and let the distance of each vertex from thecentre be a.

Then, $F_1F_2$ = 2c and AB = 2a.

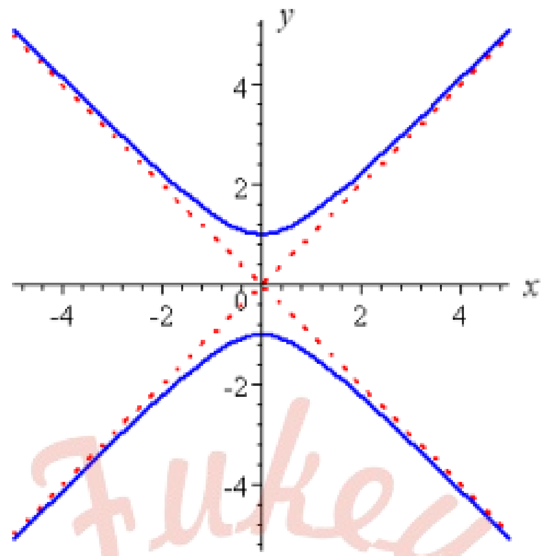
If the point P is taken at A or B, then $PF_2 - PF_1 = 2a$.

51. If the distance between the two foci is 2c, distance between two vertices is 2a, i.e. length of thetransverse axis is 2a, length of conjugate axis is 2b then a, b and c are connected as $c^2 = a^2 + b^2$

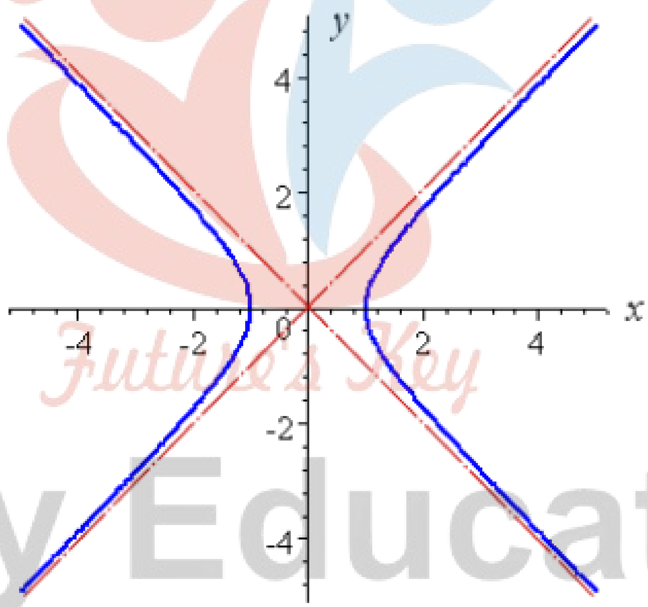



52. The ratio $e = \frac{c}{a}$ is called the eccentricity of the hyperbola. From the shape of the hyperbola, we can see that the distance of focus from origin, c is always greater than or equal to the distance of thevertex from the centre, so c is always greater than or equal to a. Because $c \geq a$, the eccentricity is never less than one.

53. The simplest hyperbola is the one in which the two axes lie along the axes and the centre is at theorigin. Two possible orientations of hyperbola are




**North–south** opening Hyperbola.




**East–west** opening Hyperbola

54.A hyperbola in which a = b is called an equilateral hyperbola.

Hyperbola is symmetric with respect to both the axes if (x, y) is a point on the hyperbola, then (−x,y), (x, −y) and (−x, −y) are also points on the hyperbola.

55.The foci are always on the transverse axis. The denominator of positive term gives the transverse axis.

56. Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of thefoci and whose end points lie on the hyperbola.

## Key Formulae

1. The general equation of second degree represents $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$a pair of straight lines if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.

   (i) a circle if $\Delta \neq 0$, $a = b$ and $h = 0$.

   (ii)a parabola if $\Delta \neq 0$, $h^2 = ab$.

   (iii)an ellipse if $\Delta \neq 0$, $h^2 < ab$.

   (iv)a hyperbola if $\Delta \neq 0$, $h^2 > ab$.

   (v)a rectangular hyperbola if $\Delta \neq 0, h^2 > ab$ and $a + b = 0$

2. The eccentricity of the conic is denoted by e.

   (i)If $e < 1$, then the conic is an ellipse.

   (ii)If $e = 1$, then the conic is a parabola.

   (iii)If $e > 1$, then the conic is a hyperbola.

   (iv)If $e = 0$, then the conic is a circle.

3. The equation of a circle with centre (h, k) and the radius r is$(x - h)^2 + (y - k)^2 = r^2$.

4. If the centre of the circle is the origin O(0, 0), then the equation of the circle reduces to $x^2 + y^2 = r^2$.

5. If the circle passes through the origin, then the equation of the circle is $x^2 + y -2hx - 2ky = 0$.

6. If the circle touches the X-axis, then the equation of the circle is $x^2 + y^2 -2hx - 2ay + h^2 = 0$.

7. If the circle touches y-axis, then the equation of the circle is $x^2 + y^2 - 2ax -2ky + k^2 = 0$.

8. If the circle touches both the axes, then the equation of the circle is $x^2 + y^2 - 2ax - 2ay + a^2 = 0$.

9. If the circle passes through the origin and centre lies on X-axis , then the equation of the circle is $x^2 + y - 2ax = 0$.

10. If the circle passes through the origin and centre lies on Y-axis , then the equation of the circle is $x^2 + y = 2ay = 0$.

11. The general equation of the circle with centre (-g, -f) and radius = $\sqrt{g^2 + f^2 - c}$ is

$x^2 + y + 2gx + 2fy + c = 0$.

12. If $g^2 + f^2 - c > 0$, then the radius of the circle is real and hence the circle is also real.

13. If $g^2 + f^2 - c = 0$, then the radius of the circle is zero and the circle is called a point circle.

14. If $g^2 + f^2 - c < 0$, then the radius of the circle is imaginary and the circle is called an imaginary circle.

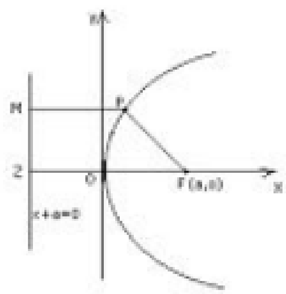
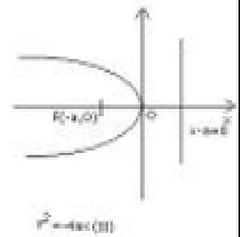
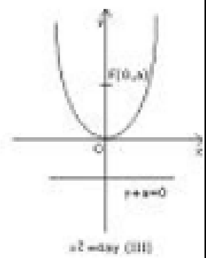
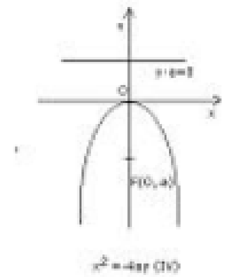
15. Let $(x_1, y_1)$ and $(x_2, y_2)$ be two given points. Then the equation of the circle with $(x_1, y_1)$ and $(x_2, y_2)$ as diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2)$.

16. Equation of the parabola in its standard form is $y^2 = 4ax$.

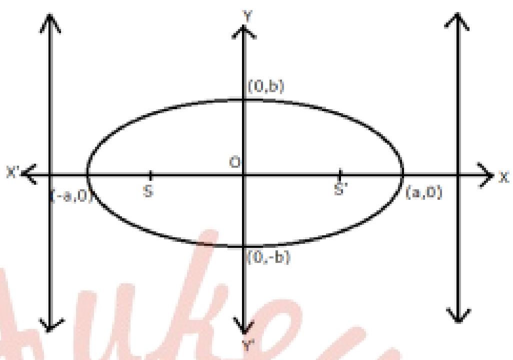
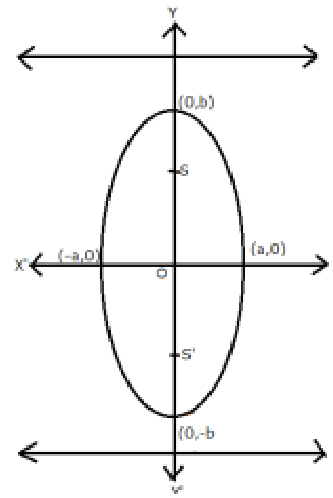
17. If the vertex of the parabola is at the point A(h, k) and its latus rectum is of length 4a, then its equation is $(y - k)^2 = 4a(x - h)$.

18. Equation of the ellipse in standard form is $\frac{x^2}{a^2} + \frac{y^2}{1} = 1$.

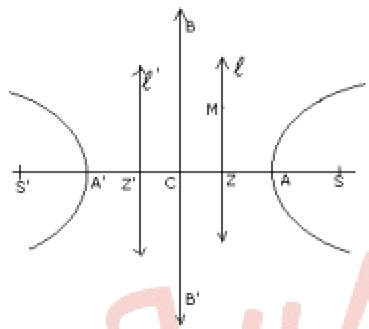
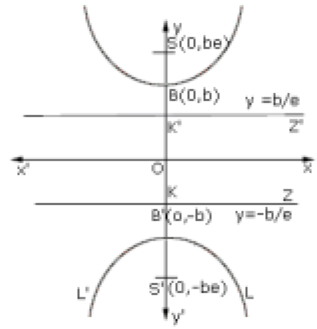
19.

| | $y^2 = 4ax$ | $y^2 = -4ax$ | $x^2 = 4ay$ | $x^2 = -4ay$ |
|---|---|---|---|---|
| Illustrations of the parabola |  |  |  |  |
| Coordinates of vertex | (0, 0) | (0,0) | (0,0) | (0,0) |
| Coordinates of focus | (a,0) | (−a,0) | (0, a) | (0, −a) |
| Equation of the directrix | $x = -a$ | $x = a$ | $y = -a$ | $y = a$ |
| Equation of the axis | $y = 0$ | $y = 0$ | $x = 0$ | $x = 0$ |
| Length of the Latus Rectum | 4a | 4a | 4a | 4a |
| Focal distance of a point P(x,y) | $a + x$ | $a - x$ | $a + y$ | $a - y$ |

20.

| | | |
|---|---|---|
| Standard equations of the Ellipse | $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ | $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$ |
| Corresponding figures of ellipse. |   |   |
| Coordinates of the centre | (0, 0) | (0, 0) |
| Coordinates of the vertices | (a, 0) and (−a, 0) | (0, +b) and (0, −b) |
| Coordinates of foci | (ae, 0) and (−ae, 0) | (0, be) and (0, −be) |
| Length of the major axis | 2a | 2b |
| Length of the minor axis | 2b | 2a |
| Equation of the major axis | y = 0 | x = 0 |
| Equation of the minor axis | x = 0 | y = 0 |
| Equations of the directrices | $x=\frac{a}{e}$ and $x=-\frac{a}{e}$ | $y=\frac{b}{e}$ and $y=-\frac{b}{e}$ |
| Eccentricity | $e=\frac{c}{a}=\sqrt{1-\frac{b^2}{a^2}}$ | $e=\frac{c}{b}=\sqrt{1-\frac{a^2}{b^2}}$ |
| Length of the latus rectum | $\frac{2b^2}{a}$ | $\frac{2a^2}{b}$ |

21.

| | | |
|---|---|---|
| Standard Equations of the Hyperbola | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ |
| Corresponding figures of Hyperbola |   |   |
| Coordinates of the centre | (0, 0) | (0, 0) |
| Coordinates of the vertices | (a, 0) and (−a, 0) | (0, b) and (0, −b) |
| Coordinates of foci | $(\pm ae, 0)$ | $(0, \pm be)$ |
| Length of the transverse axis | 2a | 2b |
| Length of the conjugate axis | 2b | 2a |
| Equations of the directrices | $x = \pm \frac{a}{e}$ | $y = \pm \frac{b}{e}$ |
| Eccentricity | $e = \frac{c}{a} = \sqrt{1 + \frac{b^2}{a^2}}$ | $e = \frac{c}{b} = \sqrt{1 + \frac{a^2}{b^2}}$ |
| Length of the latus rectum | $\frac{2b^2}{a}$ | $\frac{2a^2}{b}$ |
| Equation of the transverse axis | y = 0 | x = 0 |
| Equation of the conjugate axis | x = 0 | y = 0 |

Class : 11th Mathematics
Chapter- 11 : Conic Sections

**Conic Sections**

**Ellipse**

- An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is constant.
- The two fixed points are called the 'foci' of the ellipse.
- The midpoint of line segment joining foci is called the 'centre' of the ellipse.
- The line segment through the foci of the ellipse is called' major axis'.
- The line segment through centre & perpendicular to major axis is called minor axis.
- The end point of the major axis are called the vertices of the ellipse.
- The equation of ellipse with 'foci' on the x-axis is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$
- Length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$
- The eccentricity of an ellipse is the ratio of distances from centre of ellipse to one of foci and to one of the vertices of ellipse i.e., $e = c/a$

**Circle**

A circle is a set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is called the 'centre' of the circle and the distance from the centre to a point on the circle is called the 'radius' of the circle.

The equation of a circle with centre (h, k) and the radius r is

$$(x-h)^2+(y-k)^2=r^2$$

**Eg:** Find the equation of the circle with centre (–3, 2) and radius 4.

**Sol:** Here, h = – 3, k = 2 and r = 4

Therefore, the equation of the required circle is $(x+3)^2+(y-2)^2=16$

**Definition**

Circles, ellipses, parabolas and hyperbolas are known as conic sections because they can be obtained as intersections of plane with a double napped right circular cone.

**Hyperbola**

- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
- The equation of a hyperbola with foci on the x-axis is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
- The two fixed points are called the 'foci' of the hyperbola.
- The mid-point of the line segment joining the foci is called the 'centre' of the hyperbola.
- The line through the foci is called 'transverse axis'.
- Line through centre and perpendicular to transverse axis is called 'conjugate axis'.
- Points at which hyperbola intersects transverse axis are called 'vertices'.
- Length of the latus rectum of the hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$
- The eccentricity of a hyperbola is the ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of the hyperbola.

**parabola**

- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane. Fixed line is called 'directrix' of parabola. Fixed point F is called the 'focus'. A line through focus & perpendicular to directrix is called 'axis'. Point of intersection of parabola with axis is called 'vertex'.
- The equation of parabola with focus at (a, 0), a > 0 and directrix x= –a is $y^2=4ax$, where 4a is the length of the latus rectum

**Eg:** Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).

**Sol:** Since, vertex is at (0, 0) and focus is at (0, 2) which lies on y-axis, the y-axis is the axis of parabola. Therefore, equation of the parabola is of the form $x^2 = 4ay$. Thus we have $x^2 = 4(2)y$

i.e., $x^2 = 8y$

# Important Questions

## Multiple Choice questions-

Question 1. The straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points if

(a) $\sqrt{a^2 \times (1 + m^2)} < c$

(b) $\sqrt{a^2 \times (1 - m^2)} < c$

(c) $\sqrt{a^2 \times (1 + m^2)} > c$

(d) $\sqrt{a^2 \times (1 - m^2)} > c$

Question 2. Equation of the directrix of the parabola $x^2 = 4ay$ is

(a) $x = -a$

(b) $x = a$

(c) $y = -a$

(d) $y = a$

Question 3. The equation of parabola with vertex at origin and directrix $x - 2 = 0$ is

(a) $y^2 = -4x$

(b) $y^2 = 4x$

(c) $y^2 = -8x$

(d) $y^2 = 8x$

Question 4. The perpendicular distance from the point (3, -4) to the line $3x - 4y + 10 = 0$

(a) 7

(b) 8

(c) 9

(d) 10

Question 5. The equation of a hyperbola with foci on the x-axis is

(a) $x^2/a^2 + y^2/b^2 = 1$

(b) $x^2/a^2 - y^2/b^2 = 1$

(c) $x^2 + y^2 = (a^2 + b^2)$

(d) $x^2 - y^2 = (a^2 + b^2)$

Question 6. If the line $2x - y + \lambda = 0$ is a diameter of the circle $x^2 + y^2 + 6x - 6y + 5 = 0$ then $\lambda$ =

(a) 5

(b) 7

(c) 9

(d) 11

Question 7. The number of tangents that can be drawn from (1, 2) to $x^2 + y^2 = 5$ is

(a) 0

(b) 1

(c) 2

(d) More than 2

Question 8. The equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ will represent a real circle if

(a) $g^2 + f^2 - c < 0$

(b) $g^2 + f^2 - c \geq 0$

(c) always

(d) None of these

Question 9. The equation of parabola whose focus is (3, 0) and directrix is $3x + 4y = 1$ is

(a) $16x^2 - 9y^2 - 24xy - 144x + 8y + 224 = 0$

(b) $16x^2 + 9y^2 - 24xy - 144x + 8y - 224 = 0$

(c) $16x^2 + 9y^2 - 24xy - 144x - 8y + 224 = 0$

(d) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$

Question 10. If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latusrectum is

(a) 2/3

(b) 4/3

(c) 1/3

(d) 4

## Short Questions:

**1.** Show that the equation $x^2 + y^2 - 6x + 4y - 36 = 0$ represent a circle, also find its centre

& radius?

2. Find the equation of an ellipse whose foci are $(\pm 8, 0)$ & the eccentricity is $\frac{1}{4}$ ?
3. Find the equation of an ellipse whose vertices are $(0, \pm 10)$ & e = $\frac{4}{5}$
4. Find the equation of hyperbola whose length of latus rectum is 36 & foci are $(0, \pm 12)$
5. Find the equation of a circle drawn on the diagonal of the rectangle as its diameter, whose sides are

   $x = 6, x = -3, y = 3 \;\; \& \;\; y = -1$
6. Find the coordinates of the focus & vertex, the equations of the diretrix & the axis & length of latus rectum of the parabola x = -8y.
7. Show that the equation $6x^2 + 6y^2 + 24x - 36y - 18 = 0$ represents a circle. Also find its centre & radius.
8. Find the equation of the parabola with focus at F(5, 0) & directrix is x = -5. x = -5.
9. Find the equation of the hyperbola with centre at the origin, length of the transverse axis 18 & one focus at (0,4)
10. Find the equation of an ellipse whose vertices are $(0, \pm 13)$ & the foci are $(0, \pm 5)$

## Long Questions:

1. Find the length of major & minor axis- coordinate's of vertices & the foci, the eccentricity & length of latus rectum of the ellipse $16x^2 + y^2 = 16$
2. Find the lengths of the axis , the coordinates of the vertices & the foci the eccentricity & length of the lat us rectum of the hyperbola $25x^2 - 9y^2 = 225$.
3. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.
4. A man running in a race course notes that the sum of the distances of the two flag posts from him is always 12 m & the distance between the flag posts is 10 m. find the equation of the path traced by the man.
5. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ so that one angular point of the triangle is at the vertex of the parabola. Find the length of each side of the triangle.

# Answer Key:

## MCQ:

1. (c) $\sqrt{\{a^2 \times (1 + m^2)\}} > c$
2. (c) $y = -a$
3. (c) $y^2 = -8x$
4. (a) 7
5. (b) $x^2/a^2 - y^2/b^2 = 1$
6. (c) 9
7. (b) 1
8. (b) $g^2 + f^2 - c \geq 0$
9. (d) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$
10. (b) 4/3

## Short Answer:

1. This is of the form $x^2 + y^2 + 2gx + 2Fy + c = 0,$

where $2g = -6, 2f = 4$ &

$\therefore q = -3, f = 2 \,\&\, c = -36$

So, centre of the circle $= (-g, -f) = (3, -2)$

&

Radius of the circle $= \sqrt{q^2 + f^2 - c} = \sqrt{9 + 4 + 36}$

= 7 units

2. Let the required equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ where $a^2 > b^2$

let the foci be $(\pm c, 0), c = 8$

&

$$e = \frac{c}{a} \Leftrightarrow a = \frac{c}{e} = \frac{8}{\frac{1}{4}} = 32$$

Now $c^2 = a^2 - b^2 \Leftrightarrow b^2 = a^2 - c^2 = 1024 - 64 = 960$

$\therefore a^2 = 1024$ & $b^2 = 960$

Hence equation is $\frac{x^2}{1024} + \frac{y^2}{960} = 1$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

**3.** Let equation be

& its vertices are $(0, \pm a)$ & $a = 10$

Let

$c^2 = a^2 - b^2$

Then $e = \frac{c}{a} \Rightarrow c = ae = 10 \times \frac{4}{5} = 8$

Now

$c^2 = a^2 - b^2 \Leftrightarrow b^2 = (a^2 - c^2) = 100 - 64 = 36$

$\therefore a^2 = (10)^2 = 100$ & $b^2 = 36$

Hence the equation is

$$\frac{x^2}{36} + \frac{y^2}{100} = 1$$

**4.** Clearly C = 12

Length of cat us rectum $= 36 \Leftrightarrow \frac{2b^2}{a} = 36$

$\Rightarrow b^2 = 18a$

Now $c^2 = a^2 + b^2 \Leftrightarrow a^2 = c^2 - b^2 = 144 - 18a$

$a^2 + 18a - 144 = 0$

$(a+24)(a-6) = 0 \Leftrightarrow a = 6$ $[\because a \text{ is non negative}]$

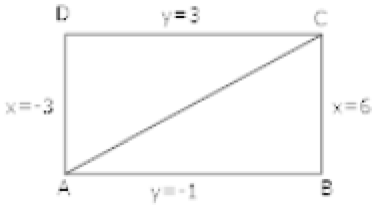
This $a^2 = 6^2 = 36$ & $b^2 = 108$

Hence, $\frac{x^2}{36} + \frac{y^2}{108} = 1$

**5.**

Let ABCD be the given rectangle & $AD = x = -3,\ BC = x = 6,\ AB = y = -1\ \&\ CD = y = -3$

Then $A(-3,-1)\ \ \&\ \ c(6,3)$




So the equation of the circle with AC as diameter is given as

$$(x+3)(x-6) + (y+1)(y-3) = 0$$

$$\Rightarrow x^2 + y^2 - 3x - 2y - 21 = 0$$

**6.**

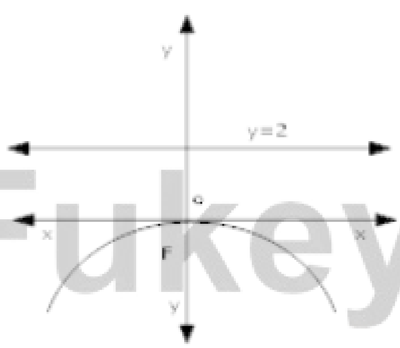
$x^2 = -8y$

& $x^2 = -4ay$

So, $4a = 8 \Leftrightarrow a = 2$

So it is case of downward parabola

o, foci is $F(0,-a)\ \ ie\ \ F(0,-2)$




Its vertex is $0(0,0)$

So, $y = a = 2$

Its axis is y – axis, whose equation is $x = 0$ length of lotus centum

$= 4a = 4 \times 2 = 8$ units.

**7.**

$$6x^2 + 6y^2 + 24x - 36y + 18 = 0$$

So $x^2 + y^2 + 4x - 6y + 3 = 0$

Where, $2g = 4, 2f = -6 \& C = 3$

$\therefore g = 2, f = -3 \& C = 3$

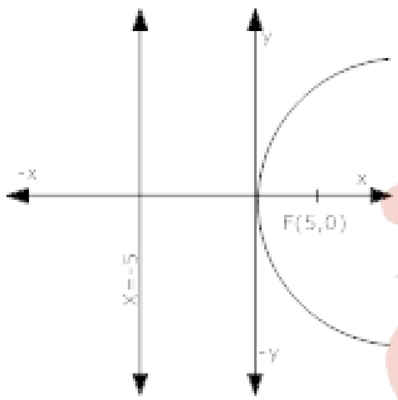
Hence, centre of circle $= (-g, -f) = (-2, 3)$

&

Radius of circle $= \sqrt{4+9+9} = \sqrt{20}$

$= 2\sqrt{5}$ units

**8.** Focus F (5, 0) lies to the right hand side of the origin




So, it is right hand parabola.

Let the required equation be

$y^2 = 4ax \quad \& \quad a = 5$

So, $y^2 = 20x$

**9.** Let its equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly, C = 4.

length of the transverse axis $= 8 \Leftrightarrow 2a = 18$

$a = 9$

Also,

$b^2 = c^2 \quad C^2 = (a^2 + b^2)$

So, $a^2 = 81 \quad \& \quad b^2 = -65$

So, equation is

**10.**Let the $\frac{y^2}{81} + \frac{x^2}{65} = 1$ equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

& a = 13

Let its foci be $(0, \pm c)$ then c = 5

$\therefore b^2 = a^2 - c^2 = 169 - 25 = 144$

So, $a^2 = 169$ & $b^2 = 144$

So, equation be $\frac{x^2}{144} + \frac{y^2}{169} = 1$

## Long Answer:

**1.** $16x^2 + y^2 = 16$

Dividing by 16,

$x^2 + \frac{y^2}{16} = 1$

So $b^2 = 1$ & $a^2 = 16$ & $b = 1$ & $a = 4$

&

$c = \sqrt{a^2 - b^2} = \sqrt{16 - 1}$

$= \sqrt{15}$

Thus $a = 4, \ b = 1$ & $c = \sqrt{15}$

**(i)** Length of major axis $= 2a = 2 \times 4 = 8$ units

Length of minor axis $= 2b = 2 \times 1 = 2$ units

**(ii)** Coordinates of the vertices are $A(-a, 0)$ & $B(a, 0)$ ie $A(-4, 0)$ & $B(4, 0)$

**(iii)** Coordinates of foci are $F_1(-c, 0)$ & $F_2(c, 0)$ ie $F_1(-\sqrt{15}, 0)$ & $F_2(\sqrt{15}, 0)$

**(iv)** Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

**(v)** Length of latus rectum $= \frac{2b^2}{a} = \frac{2}{4} = \frac{1}{2}$ units

**2.**

$25x^2 - 9y^2 = 225 \Rightarrow \frac{x^2}{9} - \frac{y^2}{25} = 1$

So, $a^2 = 9$ & $b^2 = 25$

& $c = \sqrt{a^2 + b^2} = \sqrt{9 + 25} = \sqrt{34}$

**(i)** Length of transverse axis $= 2a = 2 \times 3 = 6$ units

Length of conjugate axis $= 2b = 2 \times 5 = 10$ units

**(ii)** The coordinates of vertices are $A(-a, 0)$ & $B(a, 0)$ ie $A(-3, 0)$ & $B(3, 0)$

**(iii)** The coordinates of foci are $F_1(-c, 0)$ & $F_2(c, 0)$ ie $F_1(-\sqrt{34}, 0)$ & $F_2(\sqrt{34}, 0)$

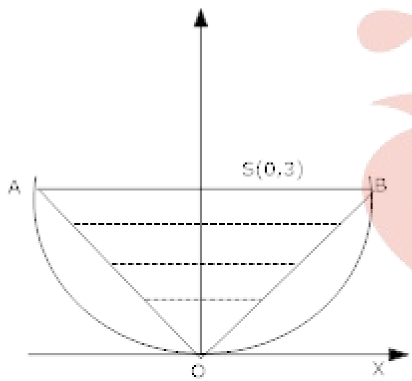
**(iv)** Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{34}}{3}$

**(v)** Length of the lat us rectum $\frac{2b^2}{a} = \frac{50}{3}$ units

**3.** The vertex of the parabola $x^2 = 12y$ ie o(0, 0)

| | | |
|---|---|---|
| 0 | 0 | 1 |
| 6 | 3 | 1 |
| -6 | 3 | 1 |

Comparing $x^2 = 12y$ with $x^2 = 4ay$ we get a = 3 the coordinates of its focus S are (0, 3).




Clearly, the ends of its latus rectum are: A(-2a, a) & B (2a, a)

Let A(-6, 3) & B(6, 3)

$\therefore$ are of $\Delta OBA = \frac{1}{2}$

$= \frac{1}{2}[1 \times (18 + 18)]$

= 18 units.

**4.** We know that on ellipse is the locus of a point that moves in such a way that the sum of its distances from two fixed points (caked foci) is constant.

So, the path is ellipse.

Let the equation of the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$

where, $b^2 = a^2 (1 - c^2)$

Clearly, $2a = 12 \;\; \& \;\; 2ae = 10$

$\Rightarrow a = b \;\; \& \;\; e = \frac{5}{6}$

$\Rightarrow b^2 = a^2\left(1-e^2\right) = 36\left(1-\frac{25}{36}\right)$

$\Rightarrow b^2 = 11$

Hence, the required equation is $\frac{x^2}{36} + \frac{y^2}{11} = 1$

**5.** Let $\Delta POR$ be an equilateral triangle inscribed in the parabola $y^2 = 4ax$

Let QP = QP = QR = PR = C

Let ABC at the x-axis at M.

Then ,

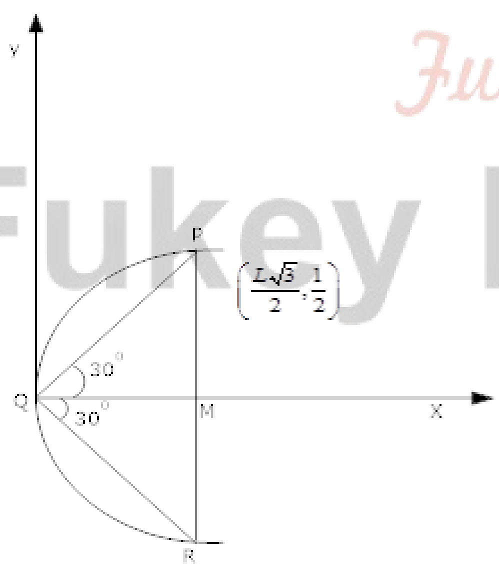
$\angle PQM = \angle RQWM = 30^\circ$

$\therefore \frac{QM}{QP} =$

$\Rightarrow \frac{L\sqrt{3}}{2}$

$\Rightarrow \frac{PM}{QP} = \sin 30^\circ \Rightarrow PM = \angle \sin 30^\circ$

$\Rightarrow \frac{L}{2}$




the coordinates of are $[\frac{L\sqrt{3}}{2}, \frac{L}{2}]$

Since P lies on the parabola $y^2 = 4ax$, we have

$$l^2 = 4a \times \frac{L\sqrt{3}}{2} \Rightarrow l = 8a\sqrt{3}$$

Hence length of each side of the triangle is $8a\sqrt{3}$ units.